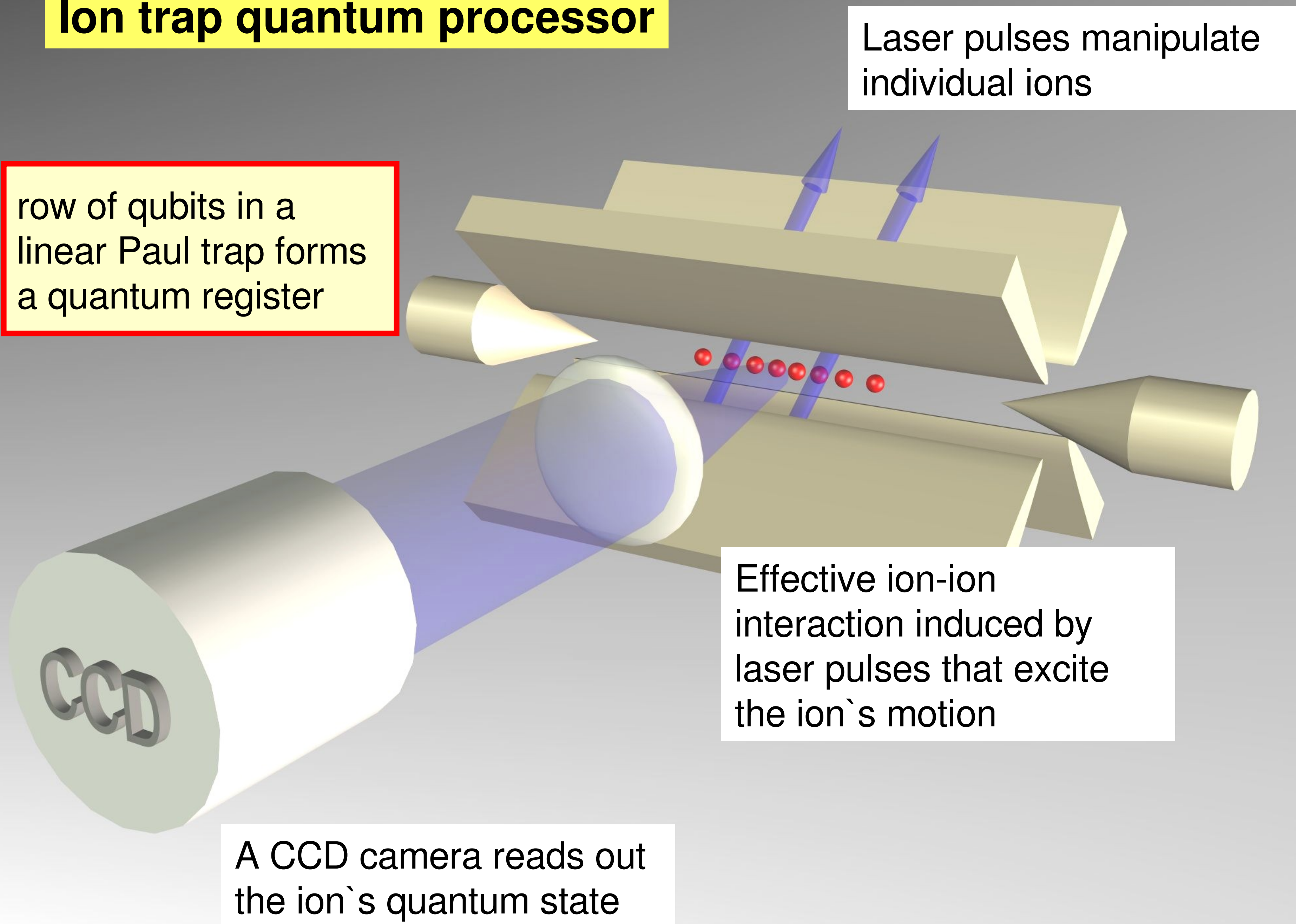
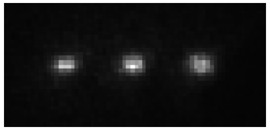
Ion trap quantum processor



Experimental setup

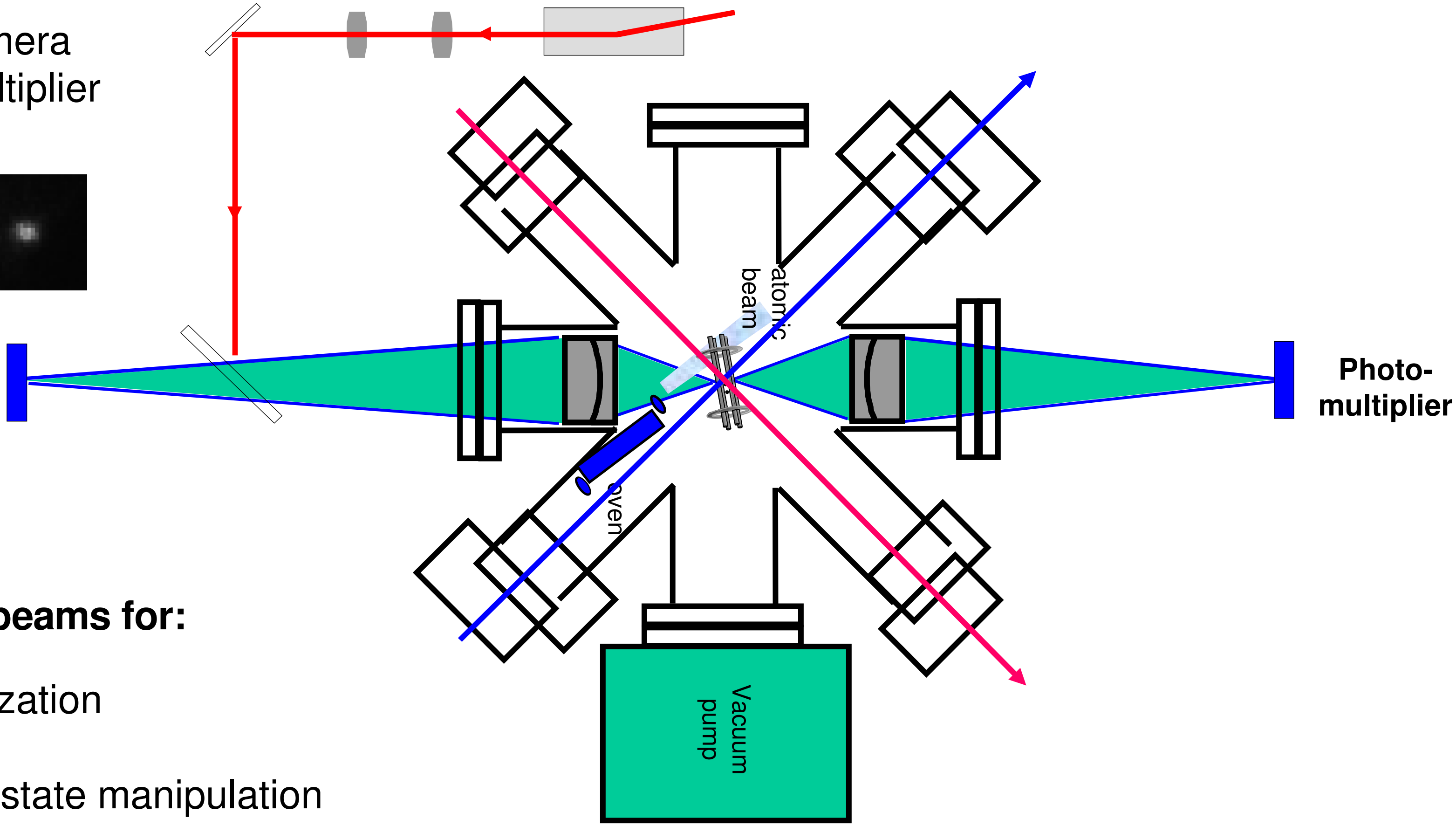
Fluorescence detection by

CCD camera
photomultiplier

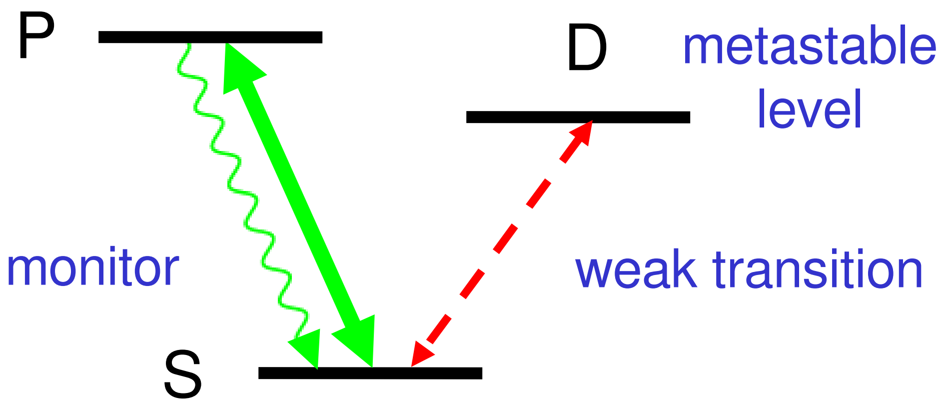


Laser beams for:

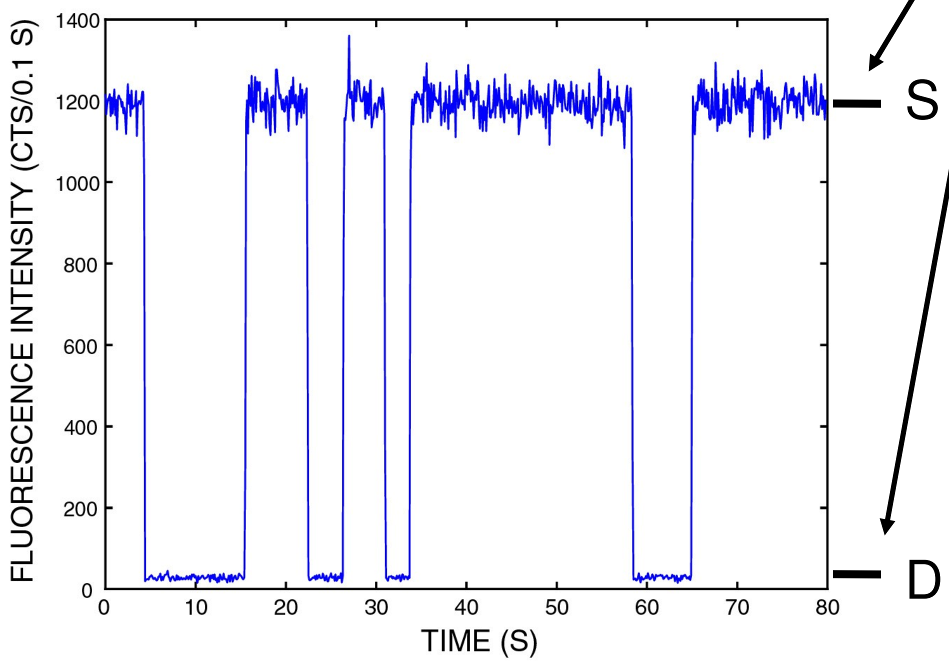
photoionization
cooling
quantum state manipulation
fluorescence excitation



Quantum jumps: spectroscopy with quantized fluorescence



absorption and emission
cause fluorescence steps
(digital quantum jump signal)

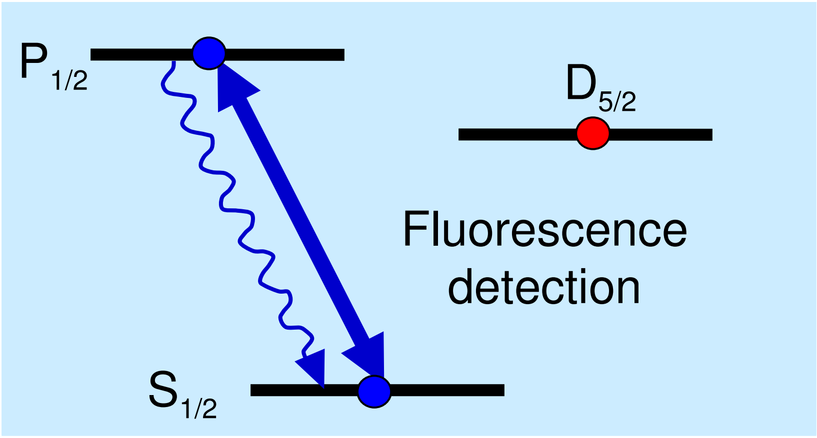


„ Quantum jump technique“
„ Electron shelving technique“

Observation of quantum jumps:

Nagourney et al., PRL **56**,2797 (1986),
Sauter et al., PRL **57**,1696 (1986),
Bergquist et al., PRL **57**,1699 (1986)

Electron shelving for quantum state detection

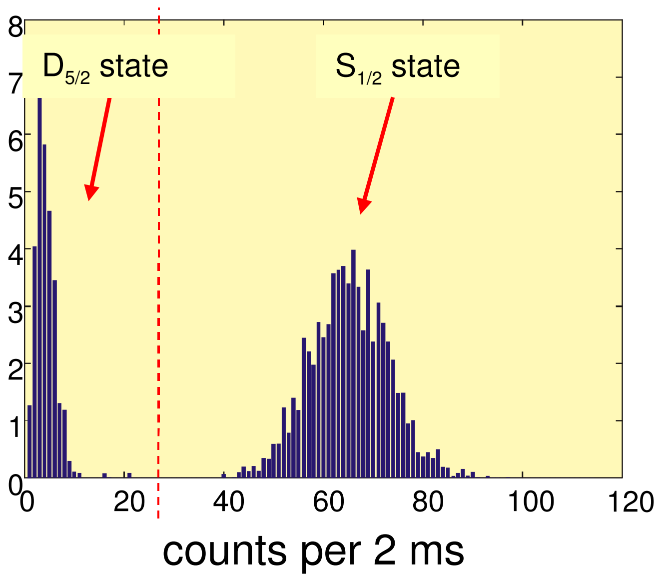


1. Initialization in a pure quantum state

2. Quantum state manipulation on $S_{1/2} – D_{5/2}$ transition

3. Quantum state measurement by fluorescence detection

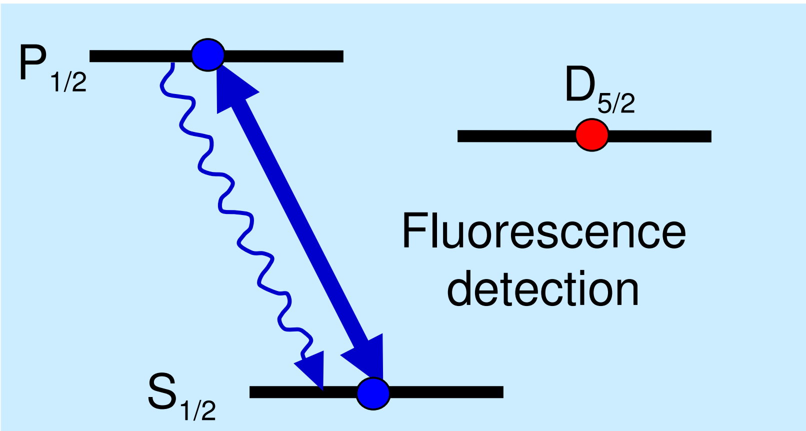
One ion : Fluorescence histogram



50 experiments / s

Repeat experiments 100-200 times

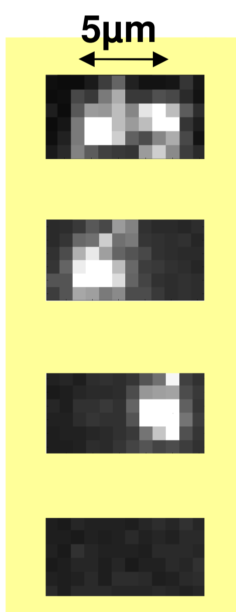
Electron shelving for quantum state detection



1. Initialization in a pure quantum state
2. Quantum state manipulation on $S_{1/2} - D_{5/2}$ transition
3. Quantum state measurement by fluorescence detection

Two ions:

Spatially resolved detection with CCD camera:



$|SS\rangle$

$|SD\rangle$

$|DS\rangle$

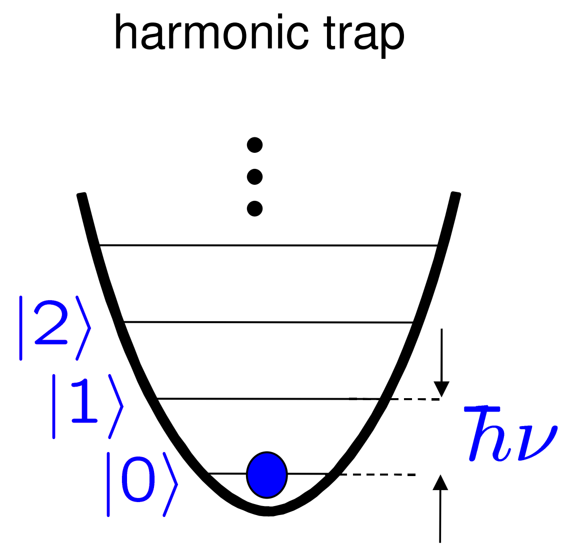
$|DD\rangle$

50 experiments / s

Repeat experiments 100-200 times

Quantum harmonical oscillator

Extension of the ground state:



$$x = \sqrt{\frac{\hbar}{2m\nu}}(a + a^\dagger)$$

$$\langle 0|x^2|0\rangle = \frac{\hbar}{2m\nu}\langle 0|(a + a^\dagger)^2|0\rangle = \frac{\hbar}{2m\nu}$$

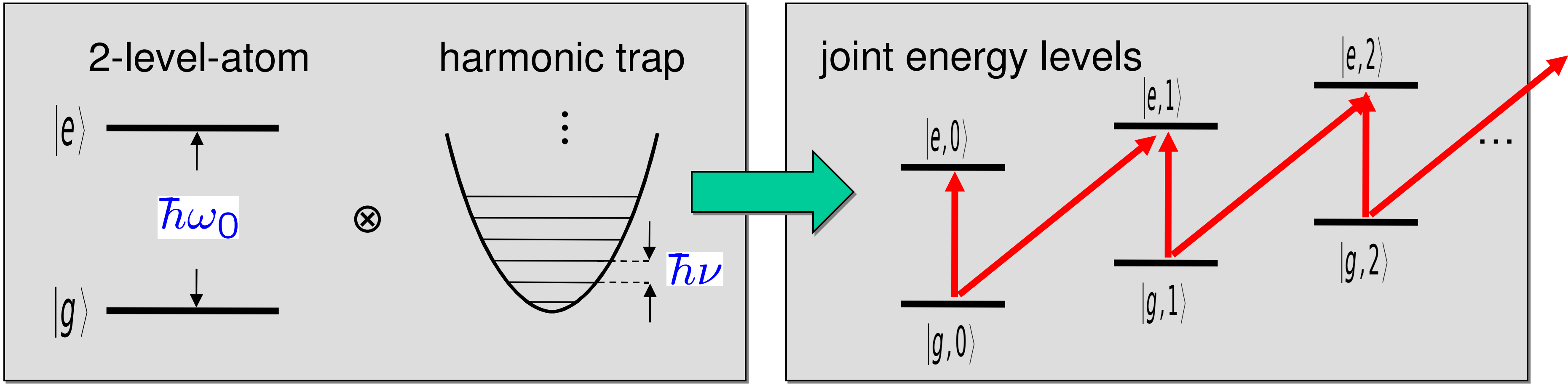
$$\left.\begin{array}{l} \nu = (2\pi)1\,\mathrm{MHz} \\ \mathrm{m{=}40\ u} \end{array}\right\} \quad \langle x^2\rangle^{1/2} = \sqrt{\frac{\hbar}{2m\nu}} \approx 11\mathrm{nm}$$

Size of the wave packet << wavelength of visible light

Energy scale of interest:

$$\hbar\nu = k_B T \longrightarrow T = \frac{\hbar\nu}{k_B} \approx 50\mu\mathrm{K}$$

Laser – ion interactions



$H^{(el)}$ $\qquad$ $H^{(m)}$

Approximations:

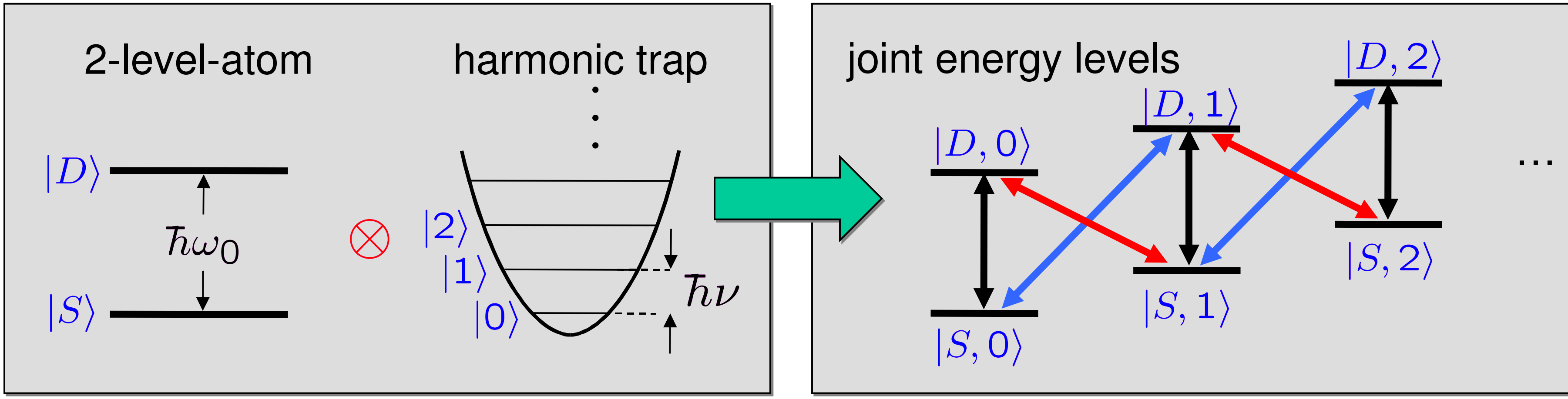
Ion: Electronic structure of the ion approximated by two-level system
(laser is (near-) resonant and couples only two levels)

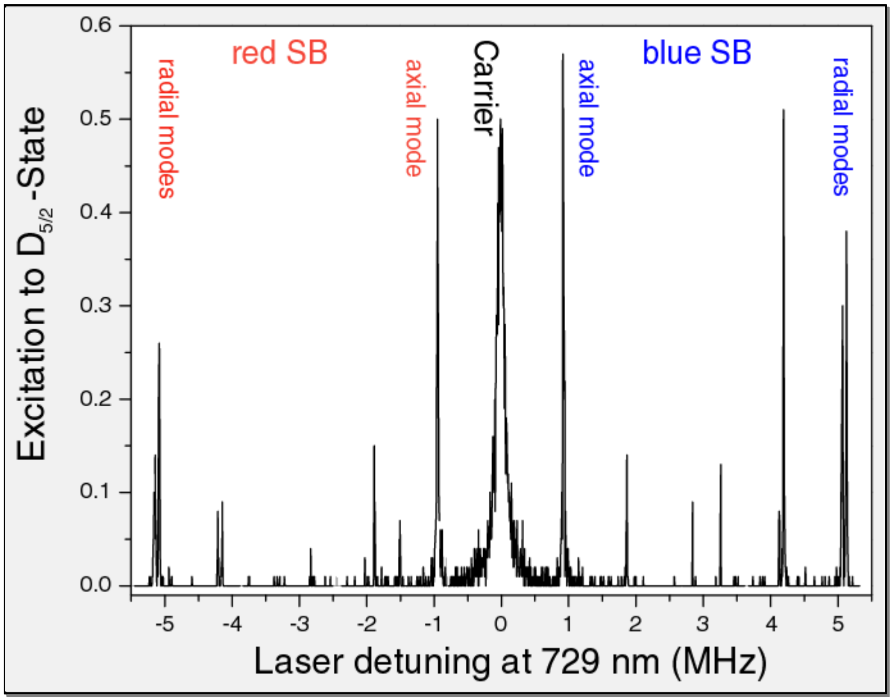
$$H^{(el)} = \hbar\frac{\omega_0}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$$

Trap: Only a single harmonic oscillator taken into account

$$H^{(m)} = \hbar\nu a^\dagger a$$

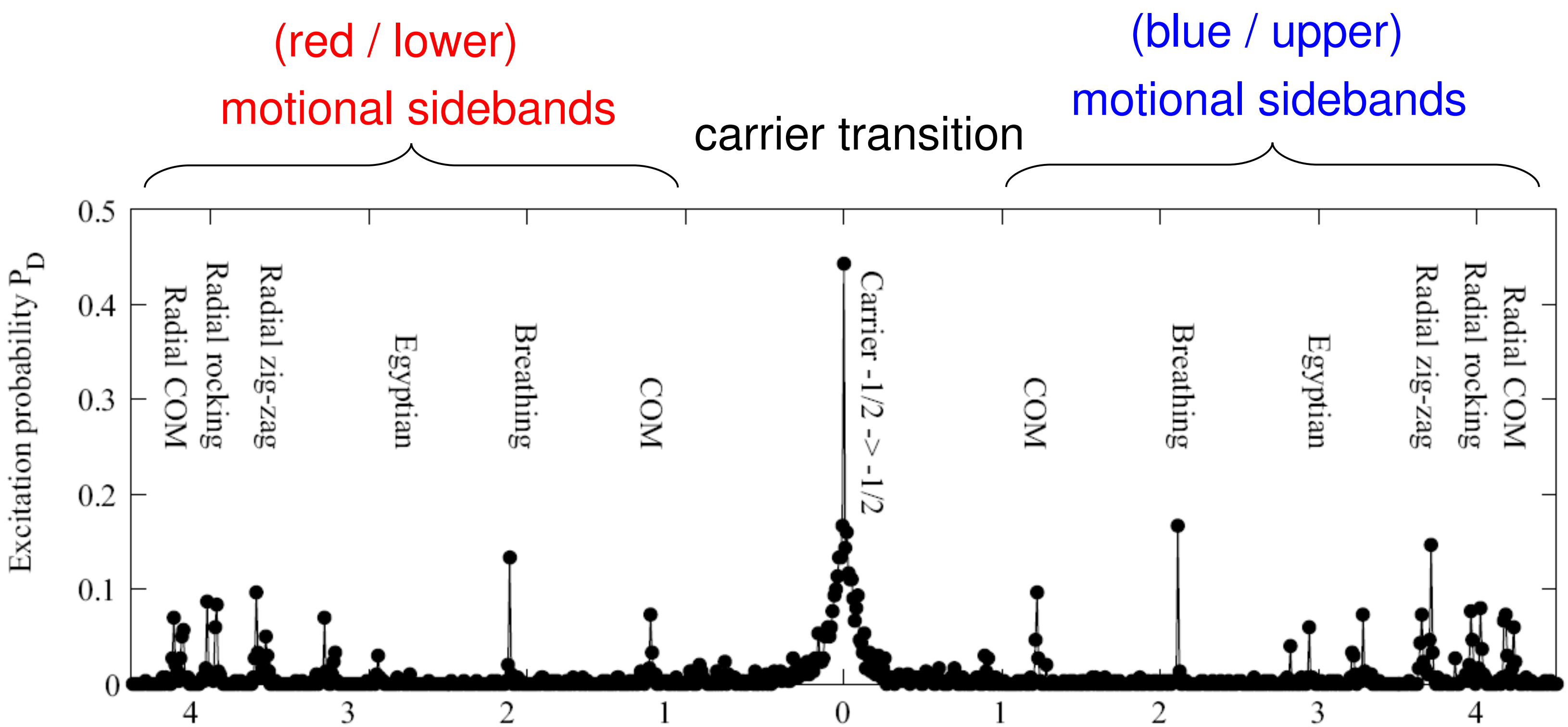
External degree of freedom: ion motion





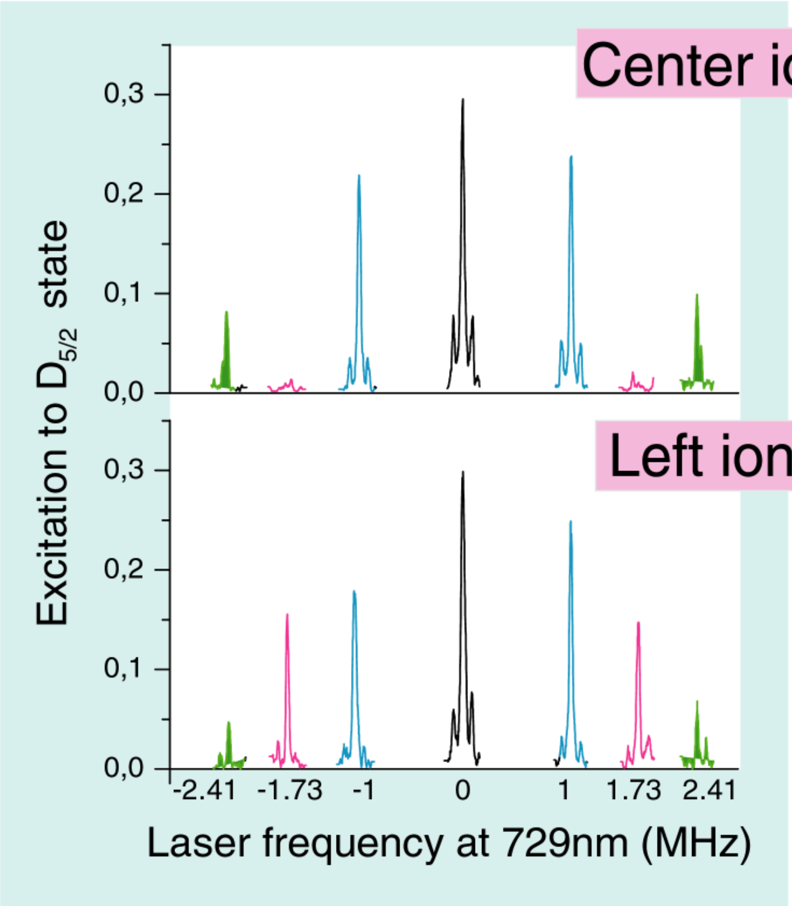
A closer look at the excitation spectrum (3 ions)

$$S_{1/2}, m = -1/2 \longleftrightarrow D_{5/2}, m = -1/2$$



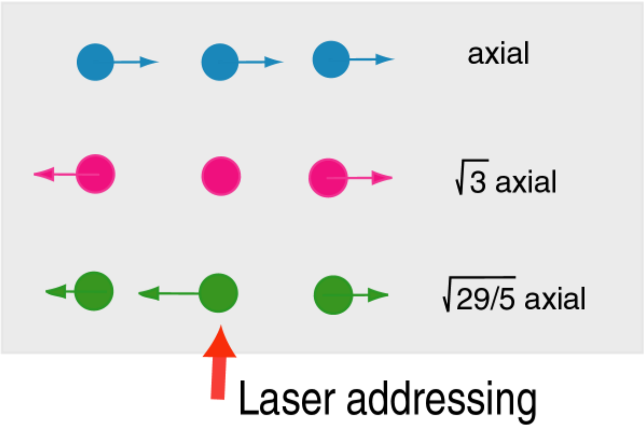
Laser detuning Δ at 729 nm (MHz)

Sideband spectra of individually addressed three ions

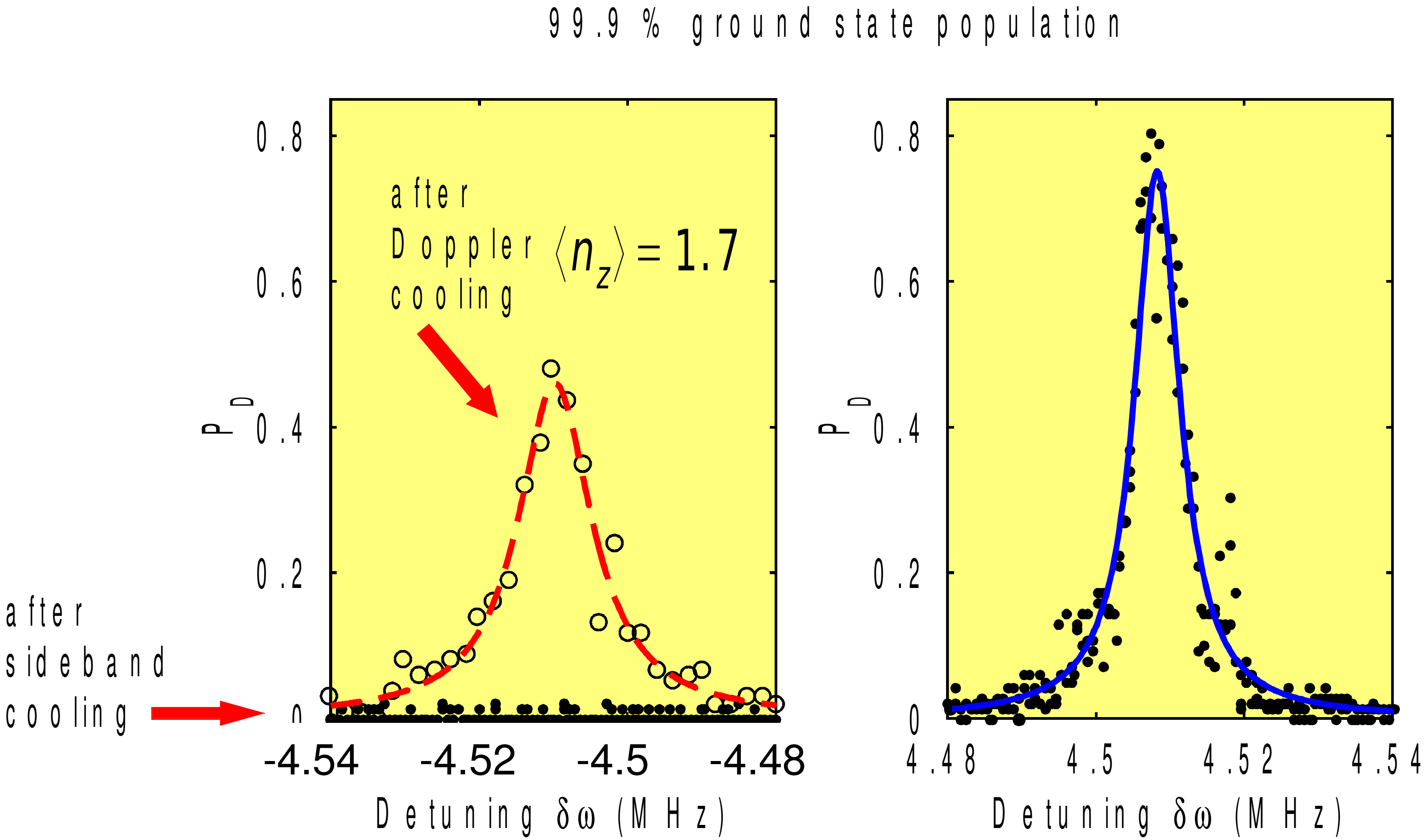


Eigen-vectors and Eigen-values

V1=	{ 0.577	0.577	0.577 }	1
V2=	{ -0.707	0	0.707 }	1.73
V3=	{ -0.408	0.817	-0.408 }	2.41



Sideband absorption spectra



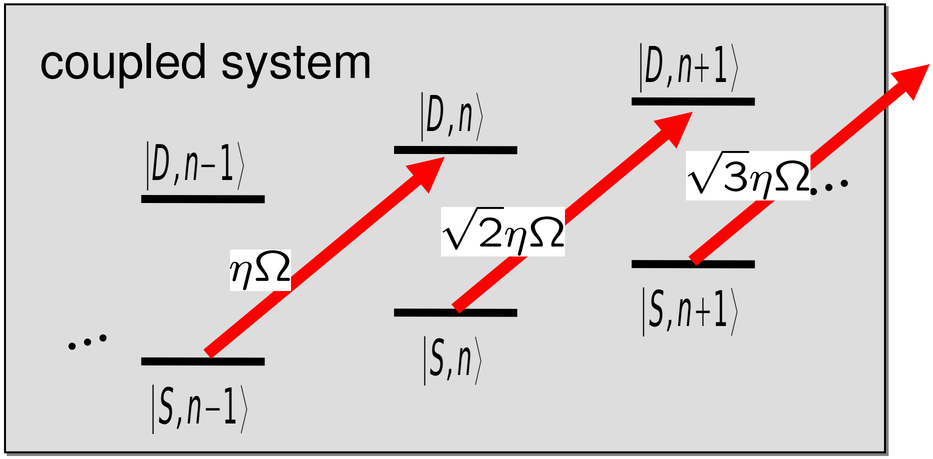
red sideband

blue sideband

Coherent excitation on the sideband

„Blue sideband" pulses:

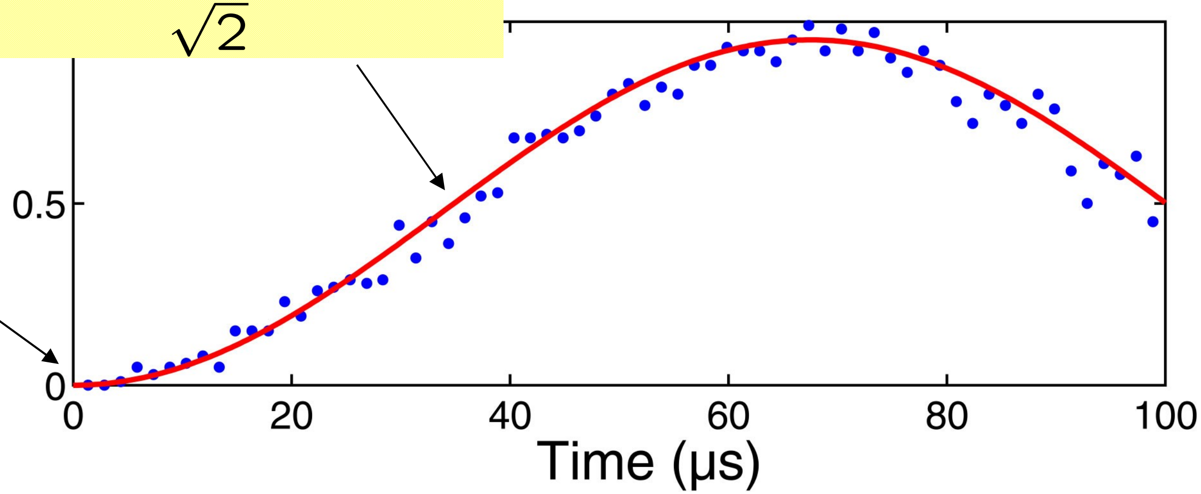
$$|S\rangle|n\rangle \longleftrightarrow |D\rangle|n+1\rangle$$



$\theta = \pi/2$: Entanglement between internal and motional state !

$$\frac{|S, n=0\rangle + |D, n=1\rangle}{\sqrt{2}}$$

$$|S, n=0\rangle$$



Single qubit operations

Arbitrary qubit rotations:

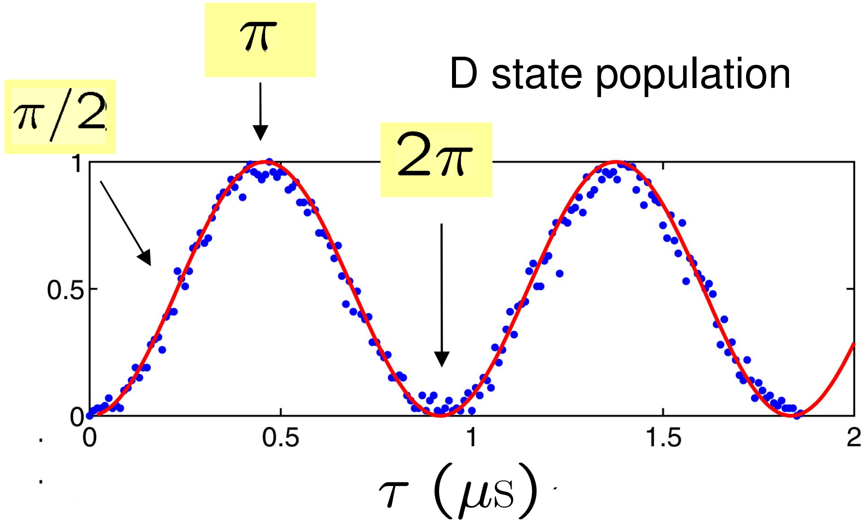
- Laser slightly detuned from carrier resonance

(z-rotations by off-resonant laser beam creating ac-Stark shifts)

or:

- Concatenation of two pulses with rotation axis in equatorial plane

Gate time : 1-10 μs ⟷ Coherence time : 2-3 ms

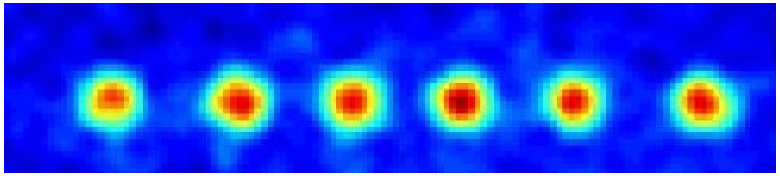


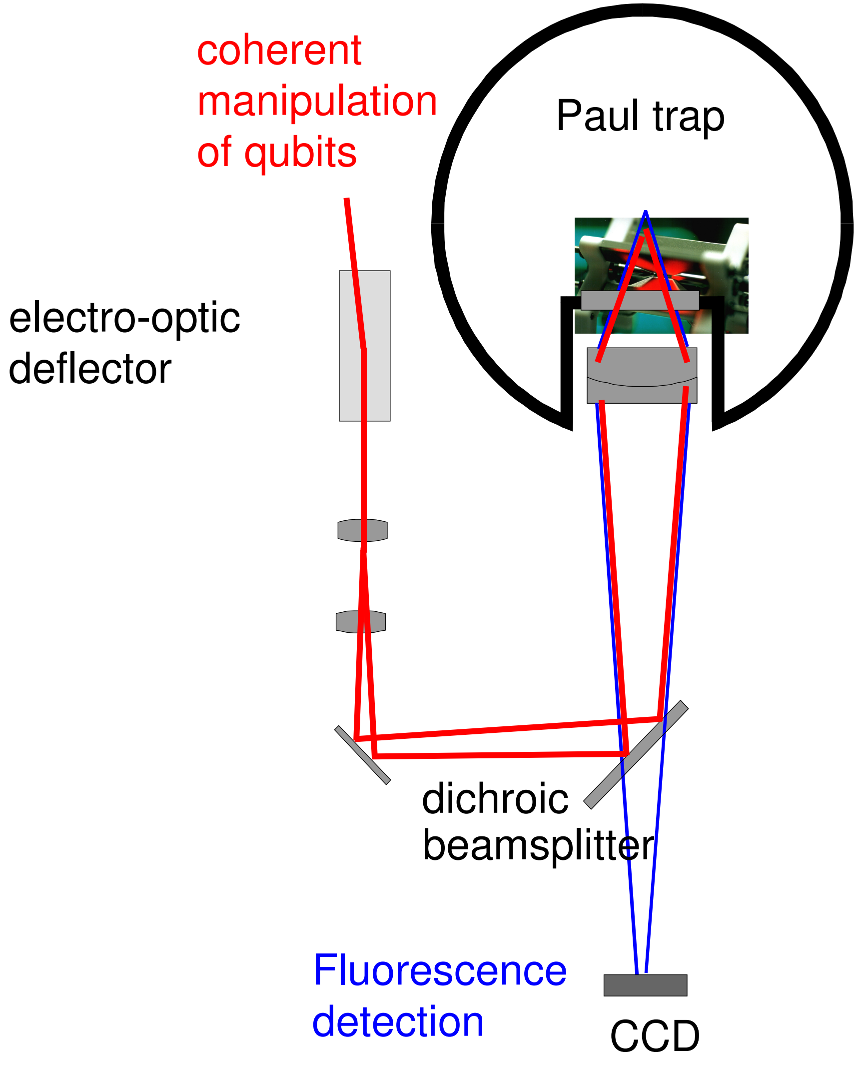
limited by

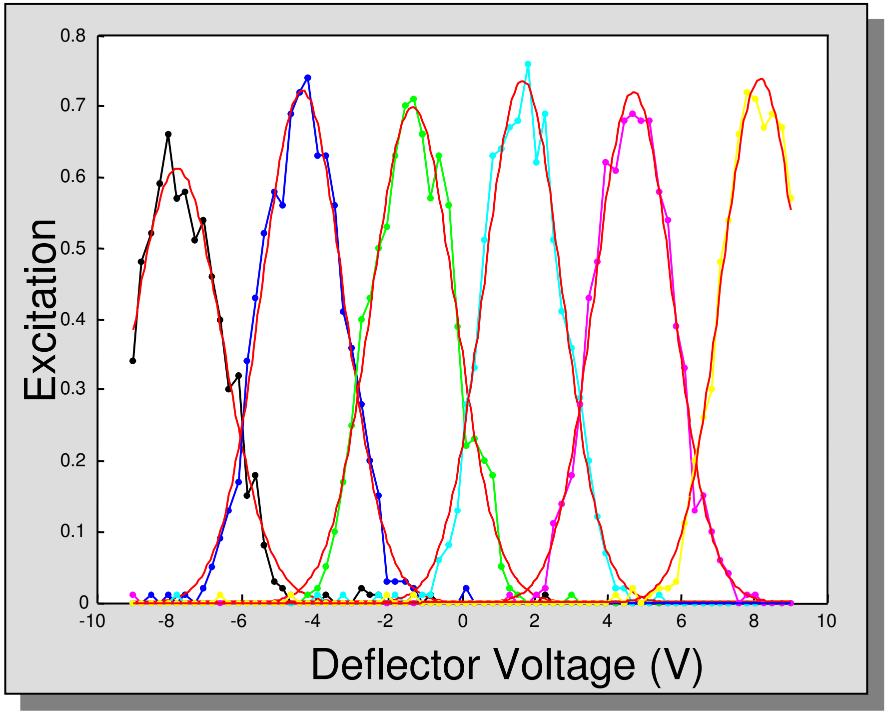
- magnetic field fluctuations
- laser frequency fluctuations

(laser linewidth δν<100 Hz)

Addressing the qubits



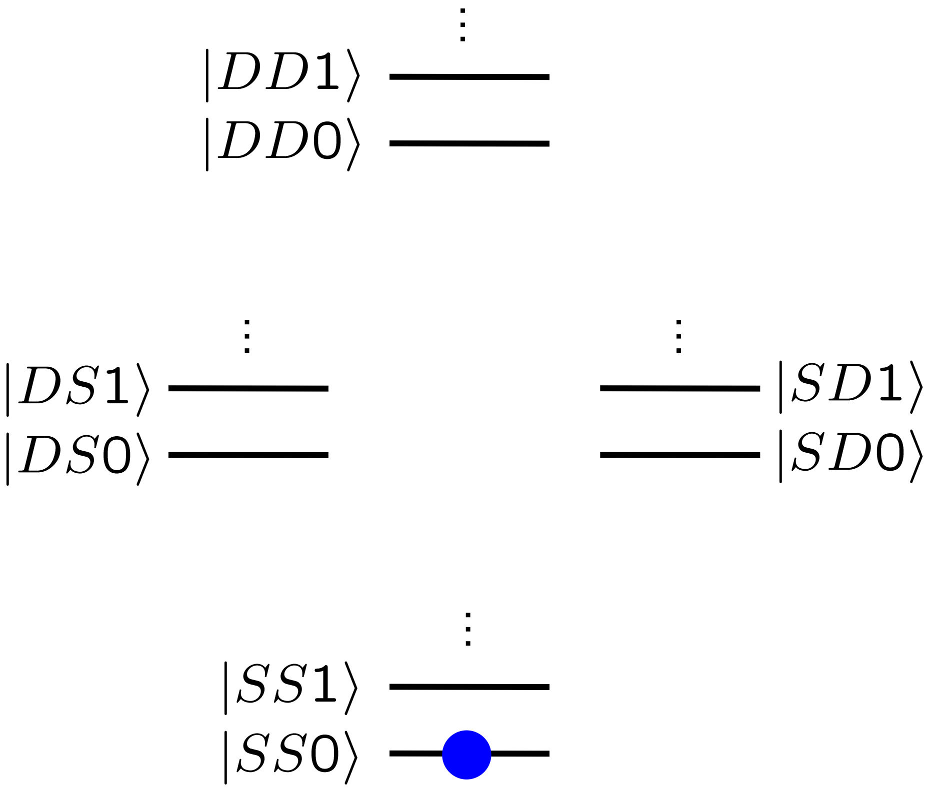




- inter ion distance: ~ 4 µm
- addressing waist: ~ 2 µm

< 0.1% intensity on neighbouring ions

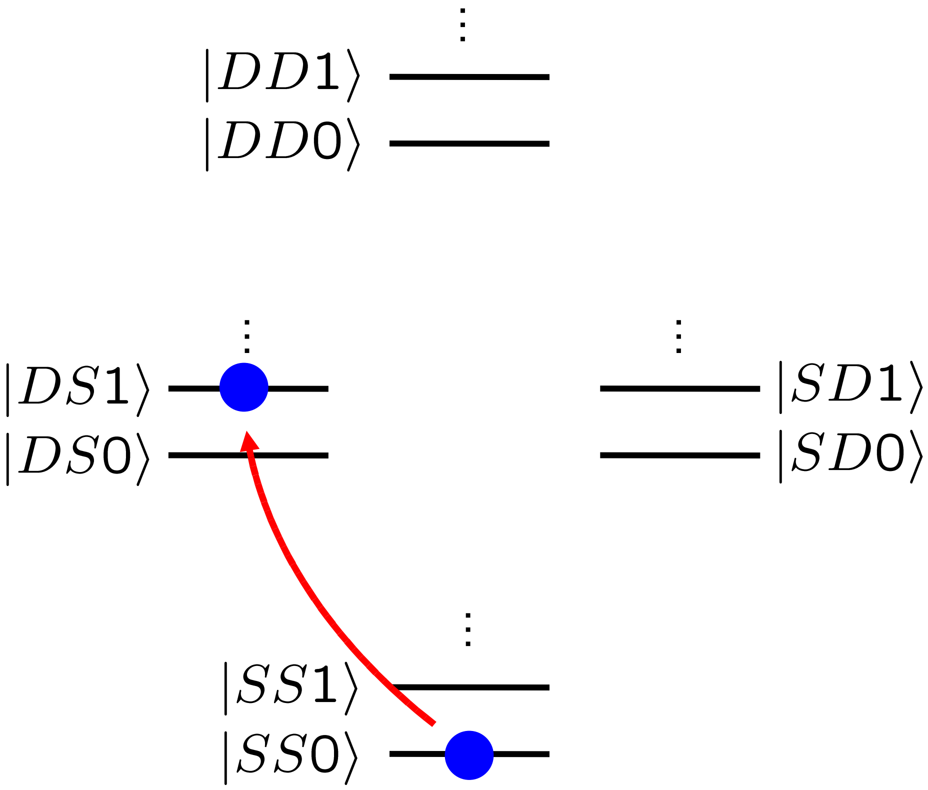
Generation of Bell states



Pulse sequence:

$|SS0\rangle$

Generation of Bell states

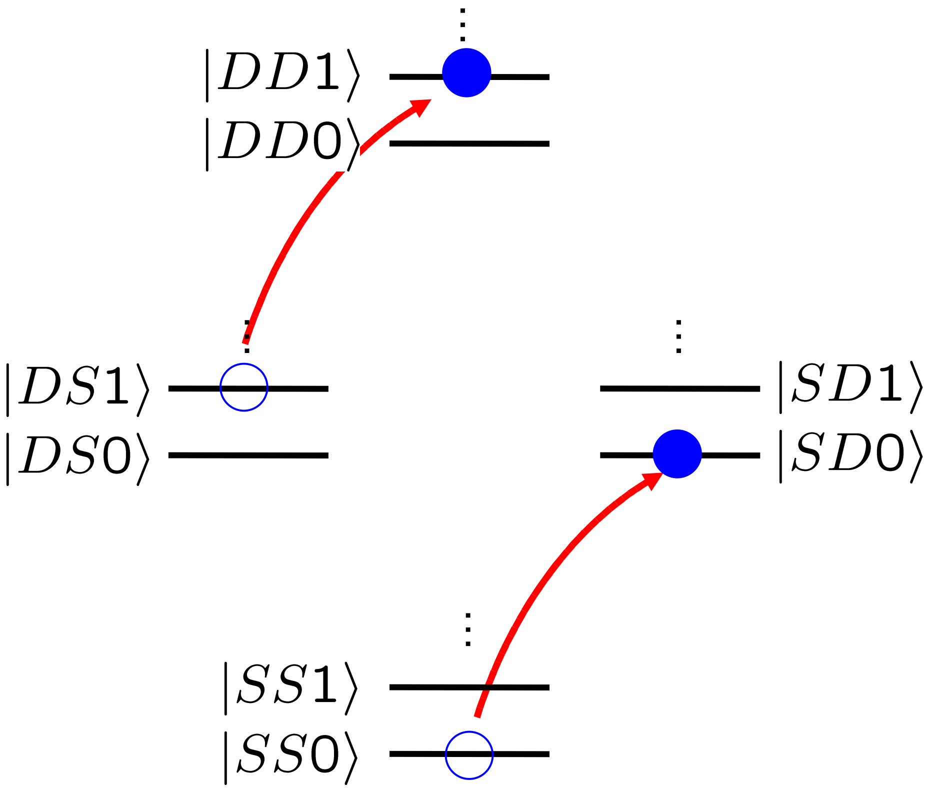


Pulse sequence:

Ion 1: $\pi/2$, blue sideband

$$|SS0\rangle + |DS1\rangle$$

Generation of Bell states



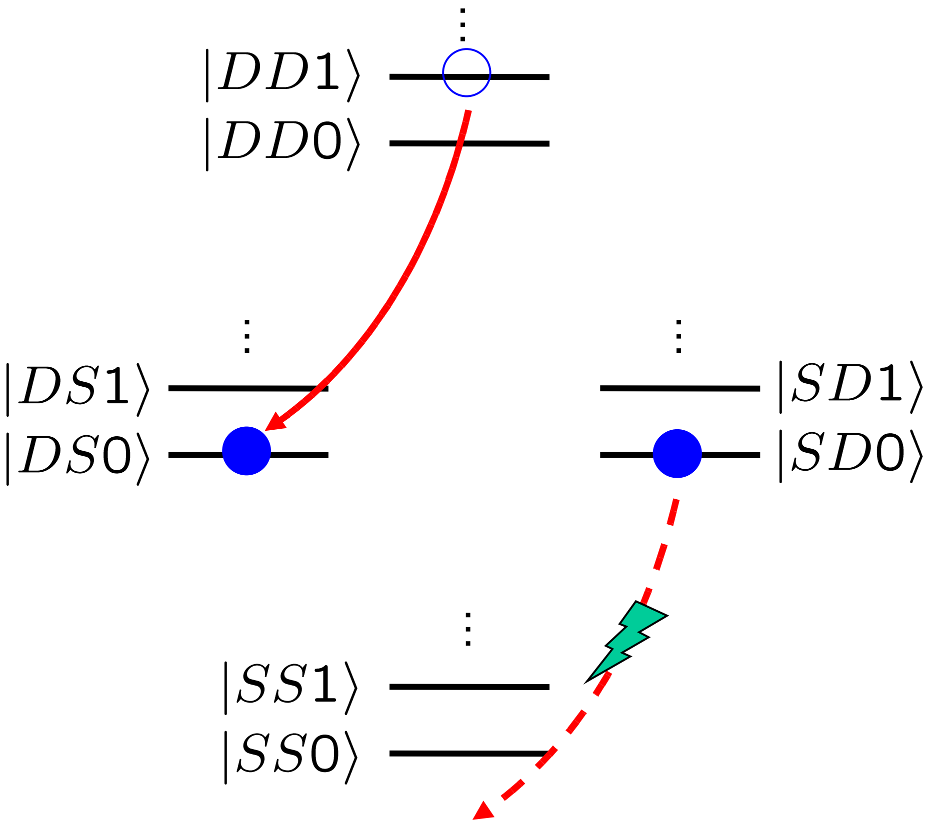
Pulse sequence:

Ion 1: π/2 , blue sideband

Ion 2: π , carrier

$$|SD0\rangle + |DD1\rangle$$

Generation of Bell states



Pulse sequence:

Ion 1: $\pi/2$, blue sideband

Ion 2: π , carrier

Ion 2: π , blue sideband

$$(|SD\rangle + |DS\rangle)|0\rangle$$

Bell state analysis

$$|SD\rangle + |DS\rangle$$

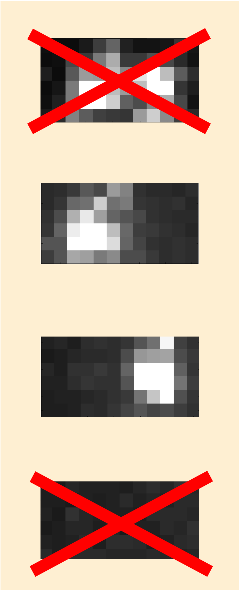
Fluorescence detection with CCD camera:

$|SS\rangle$

$|SD\rangle$

$|DS\rangle$

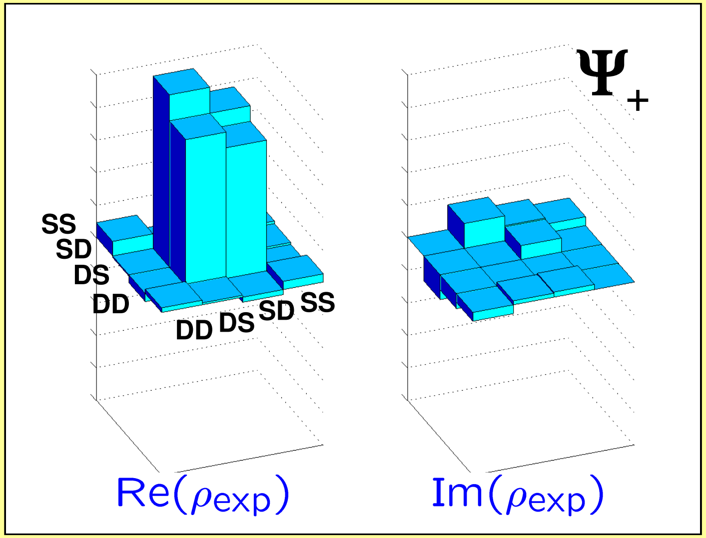
$|DD\rangle$



Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

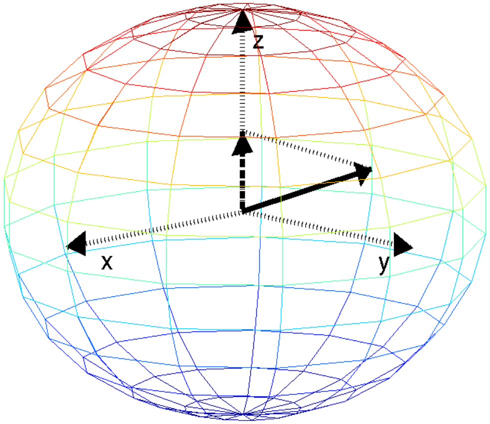
$\longrightarrow$ Measurement of the density matrix:



Obtaining a single qubits density matrix

(a naïve persons point of view)

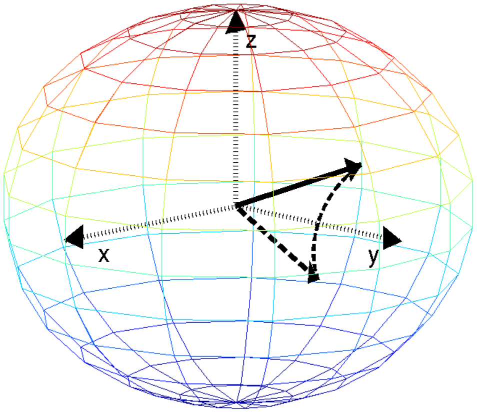
$|D\rangle$



A measurement yields the *z*-component of the Bloch vector

=> Diagonal of the density matrix

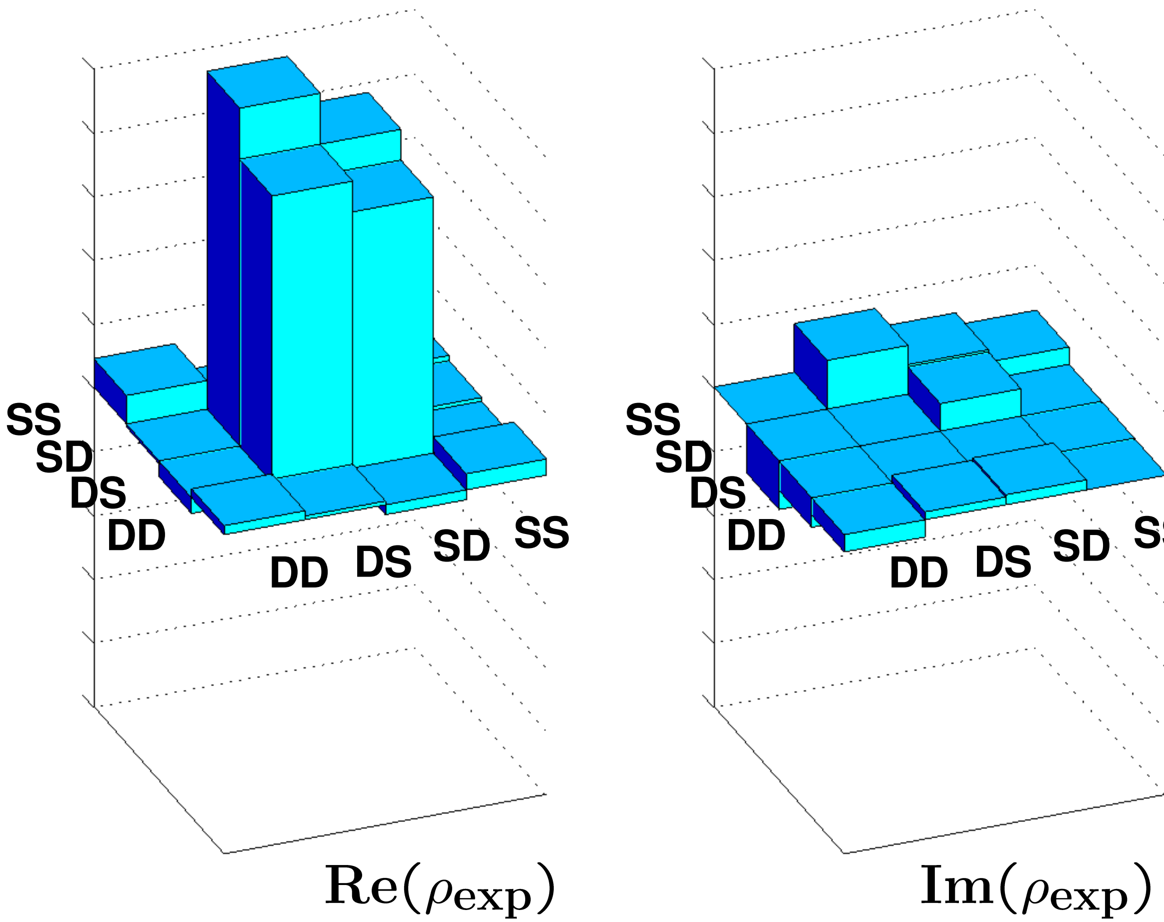
$|S\rangle$



Rotation around the *x*- or the *y*-axis prior to the measurement yields the phase information of the qubit.

=> coherences of the density matrix

Bell state reconstruction



$|SD\rangle + |DS\rangle$

F=0.91

Phase gate ⇔ CNOT

$$
\begin{array}{cccc}
 & \xrightarrow{R_1^C(\frac{\pi}{2},\frac{\pi}{2})} & \xrightarrow{\text{Phasegate}} & \xrightarrow{R_1^C(\frac{\pi}{2},-\frac{\pi}{2})} \\
\hline
|0\rangle \otimes |0\rangle & |0\rangle \otimes (|0\rangle + |1\rangle) & |0\rangle \otimes (|0\rangle + |1\rangle) & |0\rangle \otimes |0\rangle \\
|0\rangle \otimes |1\rangle & |0\rangle \otimes (|0\rangle - |1\rangle) & |0\rangle \otimes (|0\rangle - |1\rangle) & |0\rangle \otimes |1\rangle \\
|1\rangle \otimes |0\rangle & |1\rangle \otimes (|0\rangle + |1\rangle) & |1\rangle \otimes (|0\rangle - |1\rangle) & |1\rangle \otimes |1\rangle \\
|1\rangle \otimes |1\rangle & |1\rangle \otimes (|0\rangle - |1\rangle) & |1\rangle \otimes (|0\rangle + |1\rangle) & |1\rangle \otimes |0\rangle
\end{array}
$$

Both, the phase gate as well the CNOT gate can be converted into each other with single qubit operations.

$$R^C(\pi/2, \pi/2) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$R^C(\pi/2, -\pi/2) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

Together with the three single qubit gates, we can implement any unitary operation!

Quantum gate proposals with trapped ions

VOLUME 74, NUMBER 20 PHYSICAL REVIEW LETTERS 15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universiät Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

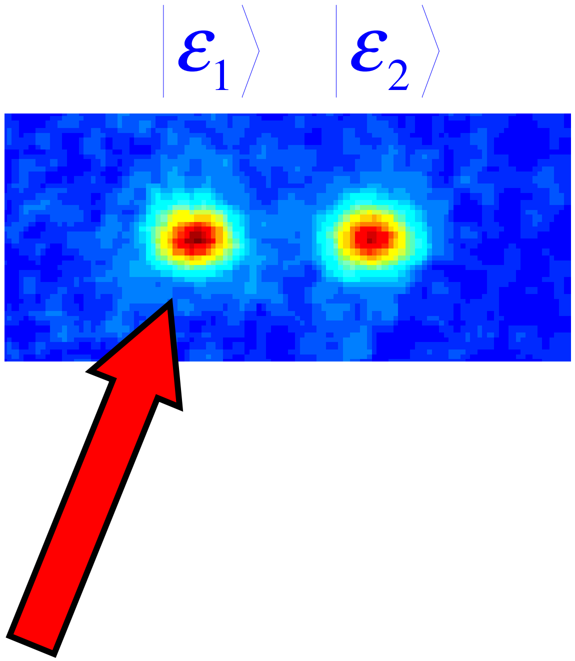
PACS numbers: 89.80.+h, 03.65.Bz, 12.20.Fv, 32.80.Pj

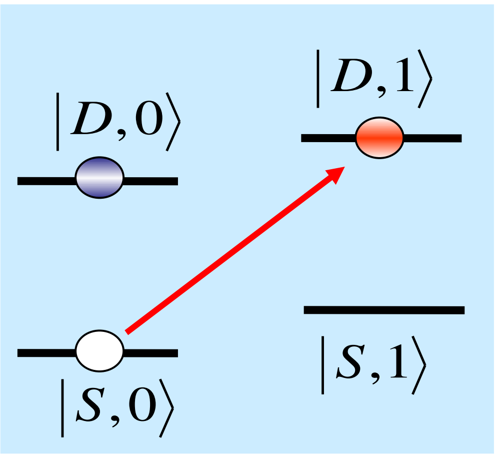
...allows the realization of a ***universal*** quantum computer !

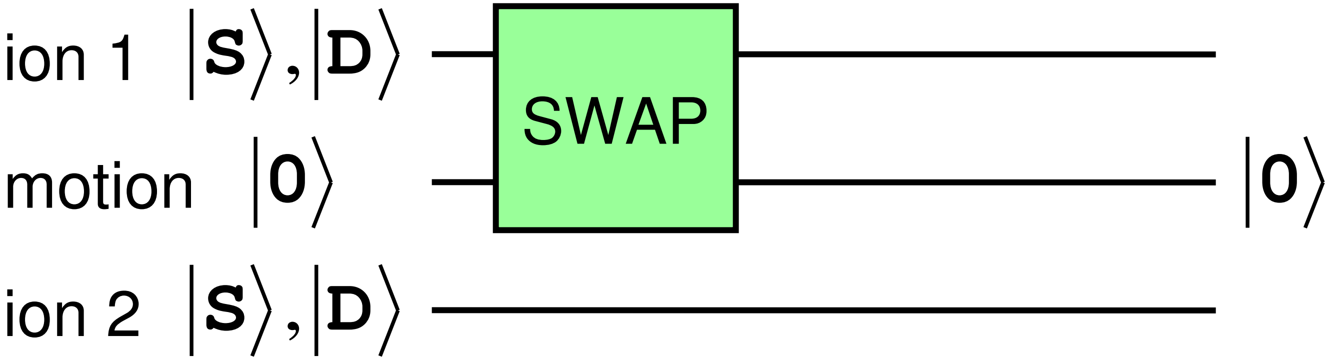
Some other gate proposals by:

- Cirac & Zoller
- Mølmer & Sørensen, Milburn
- Jonathan, Plenio & Knight
- Geometric phases
- Leibfried & Wineland

Cirac - Zoller two-ion phase gate

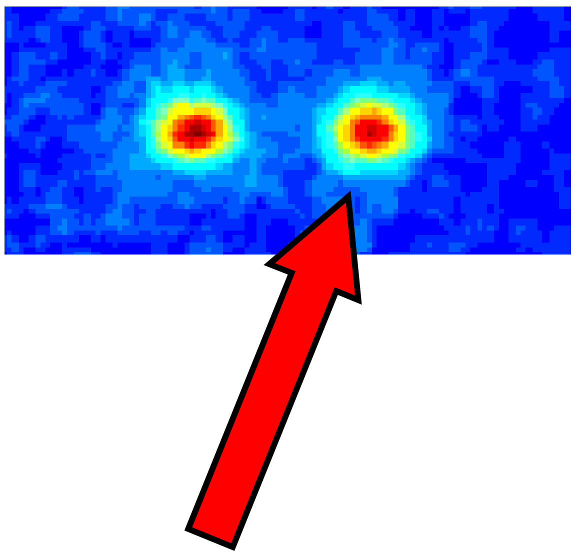




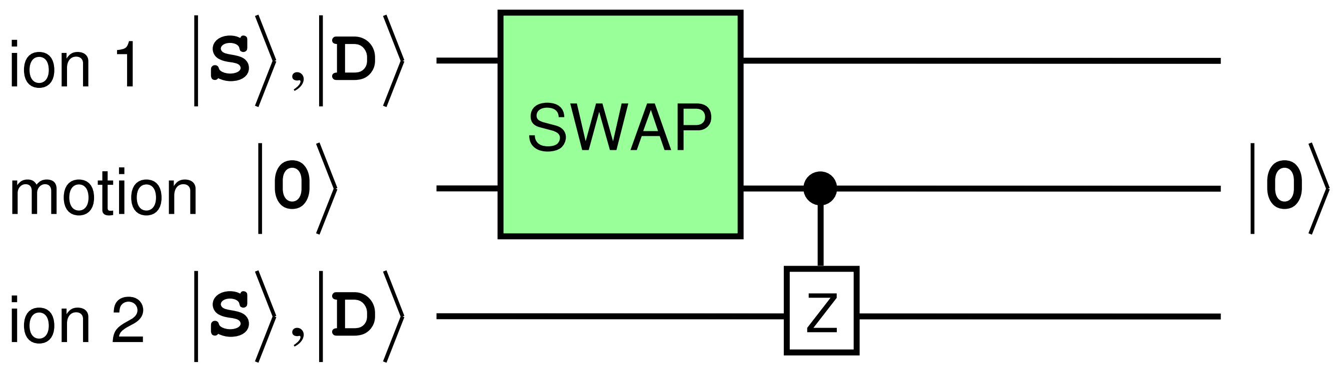


Cirac - Zoller two-ion phase gate

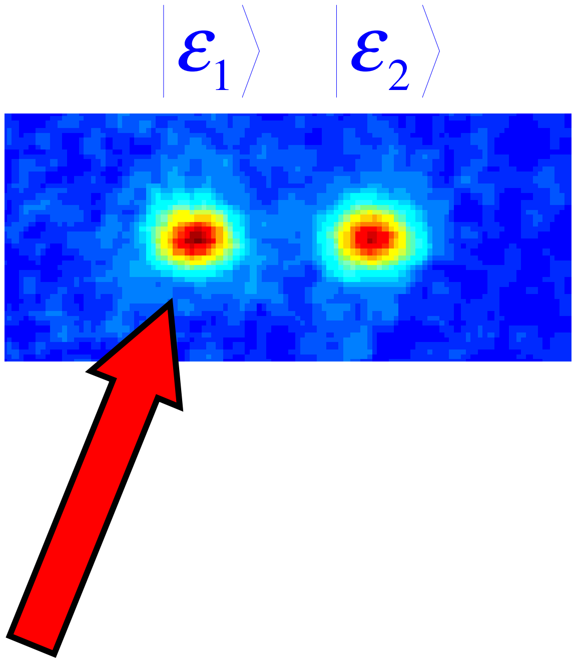
$|\mathcal{E}_1\rangle$ $|\mathcal{E}_2\rangle$

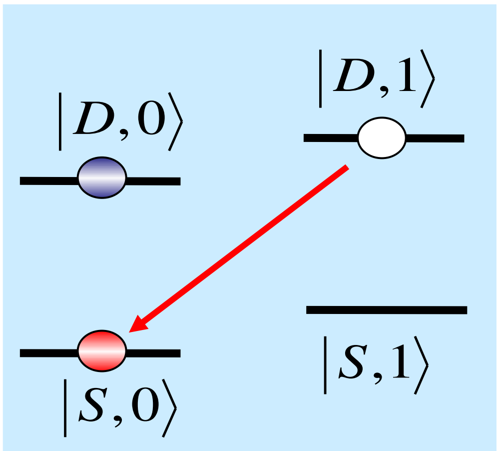


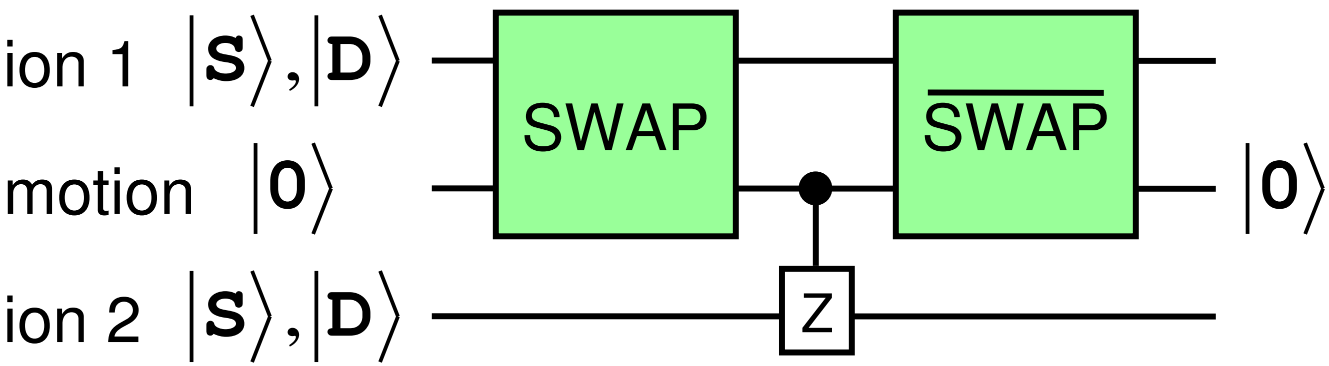
Phase gate using the motion and the target bit.



Cirac - Zoller two-ion phase gate

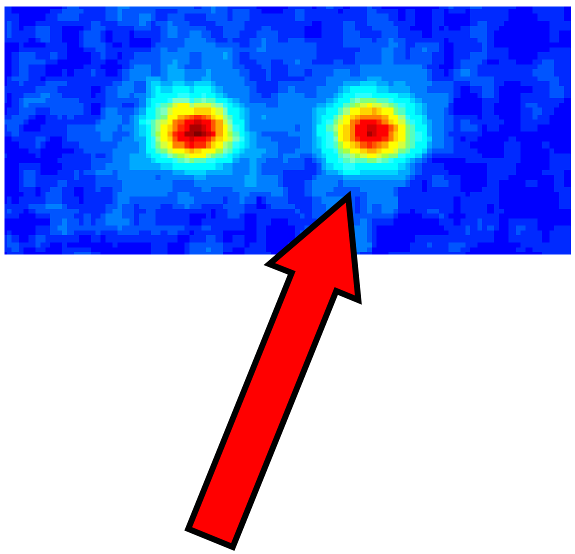




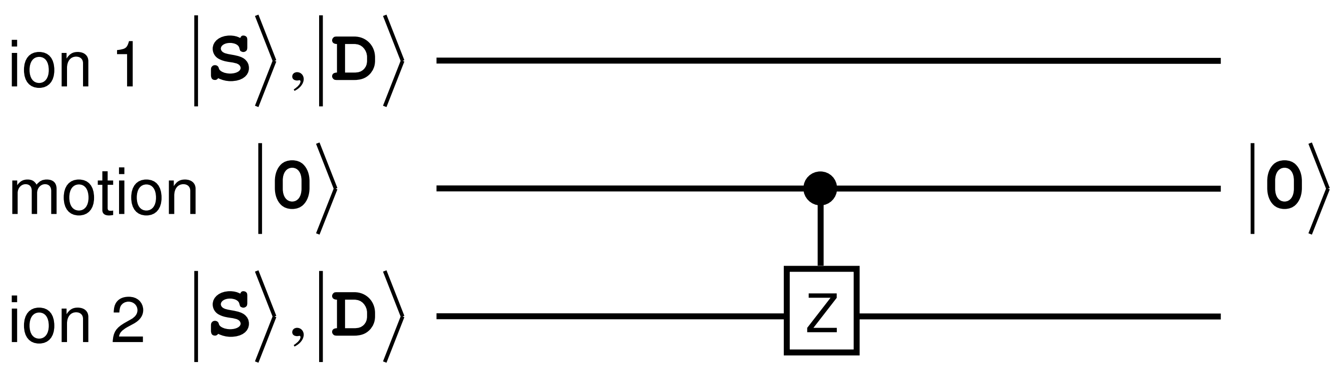


Cirac - Zoller two-ion phase gate

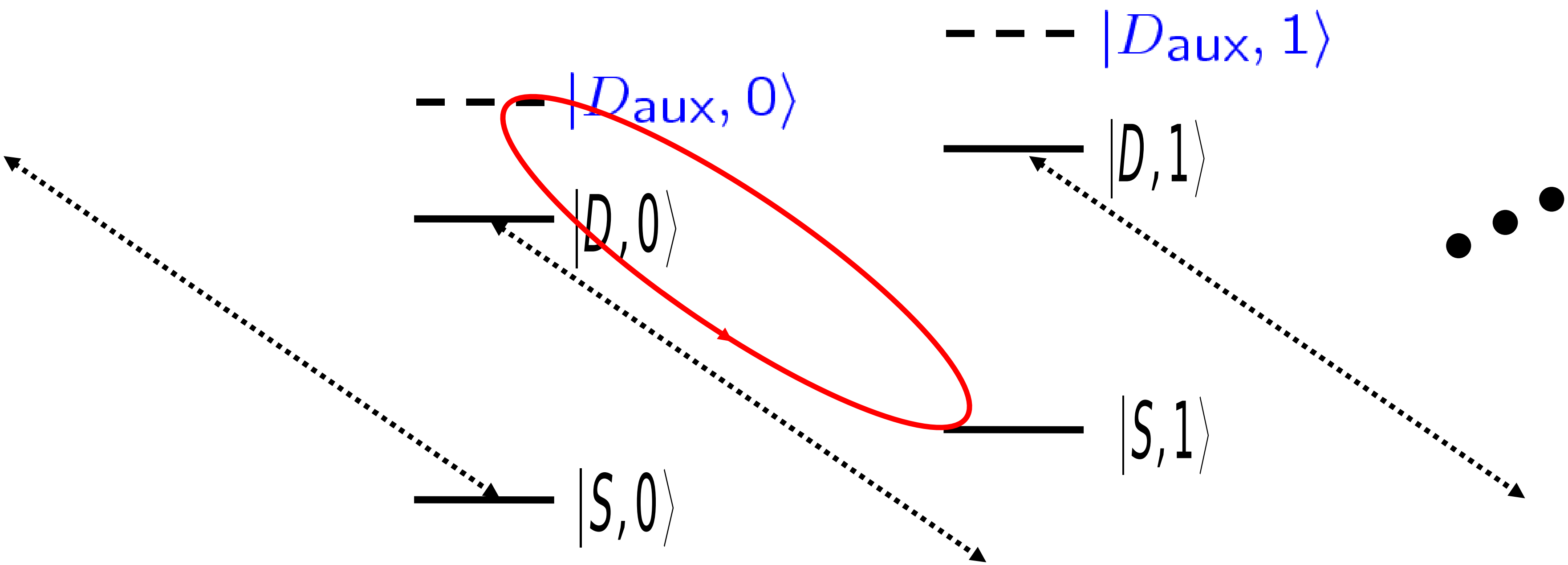
$|\varepsilon_1\rangle$ $|\varepsilon_2\rangle$



Phase gate using the motion and the target bit.



Cirac-Zoller phase gate: the key step



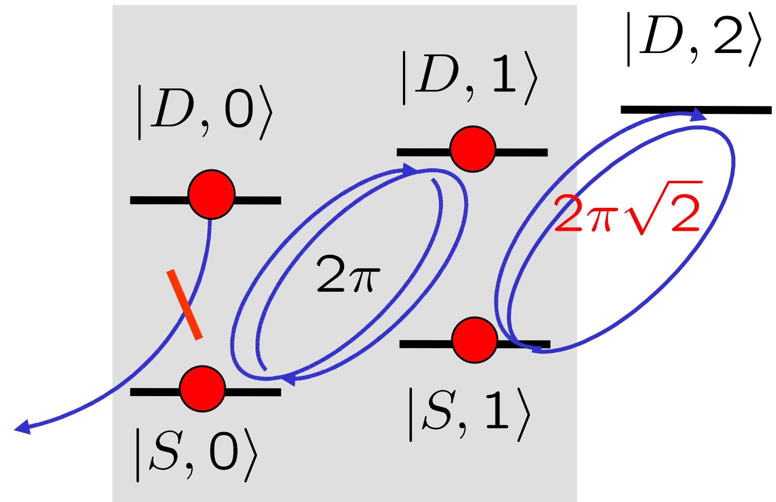
A 2π pulse is applied to only one of ion-(crystal)s states => only one states acquires a phase factor of –1.

An additional Zeeman level can be used as the auxilary state.

=> gate is sensitive to magnetic field fluctuations!

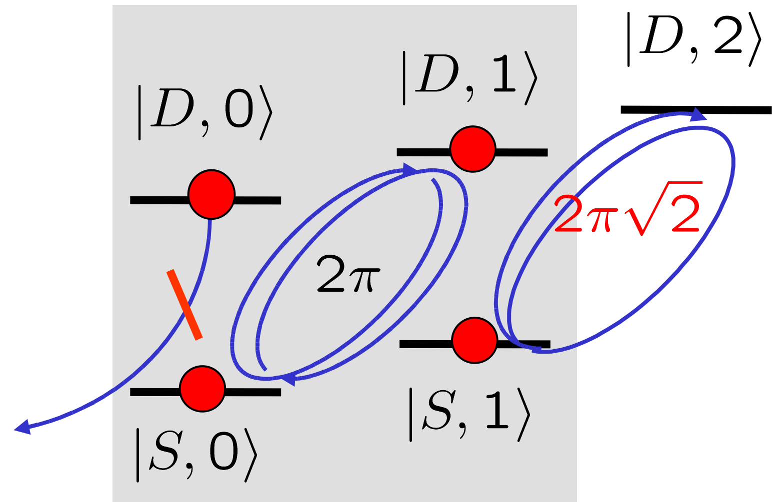
How do you do with just a two-level system?

$$U_\Phi = \begin{array}{c} \begin{array}{cccc} |D,0\rangle & |S,0\rangle & |D,1\rangle & |S,1\rangle \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & \boxed{?} \end{pmatrix} \end{array}$$

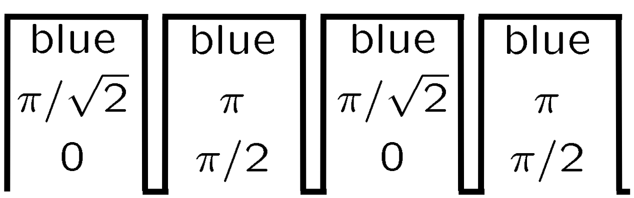


Phase gate

$$
\begin{array}{cc}
 & \begin{array}{cccc} |D,0\rangle & |S,0\rangle & |D,1\rangle & |S,1\rangle \end{array} \\
U_\Phi = & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}
\end{array}
$$



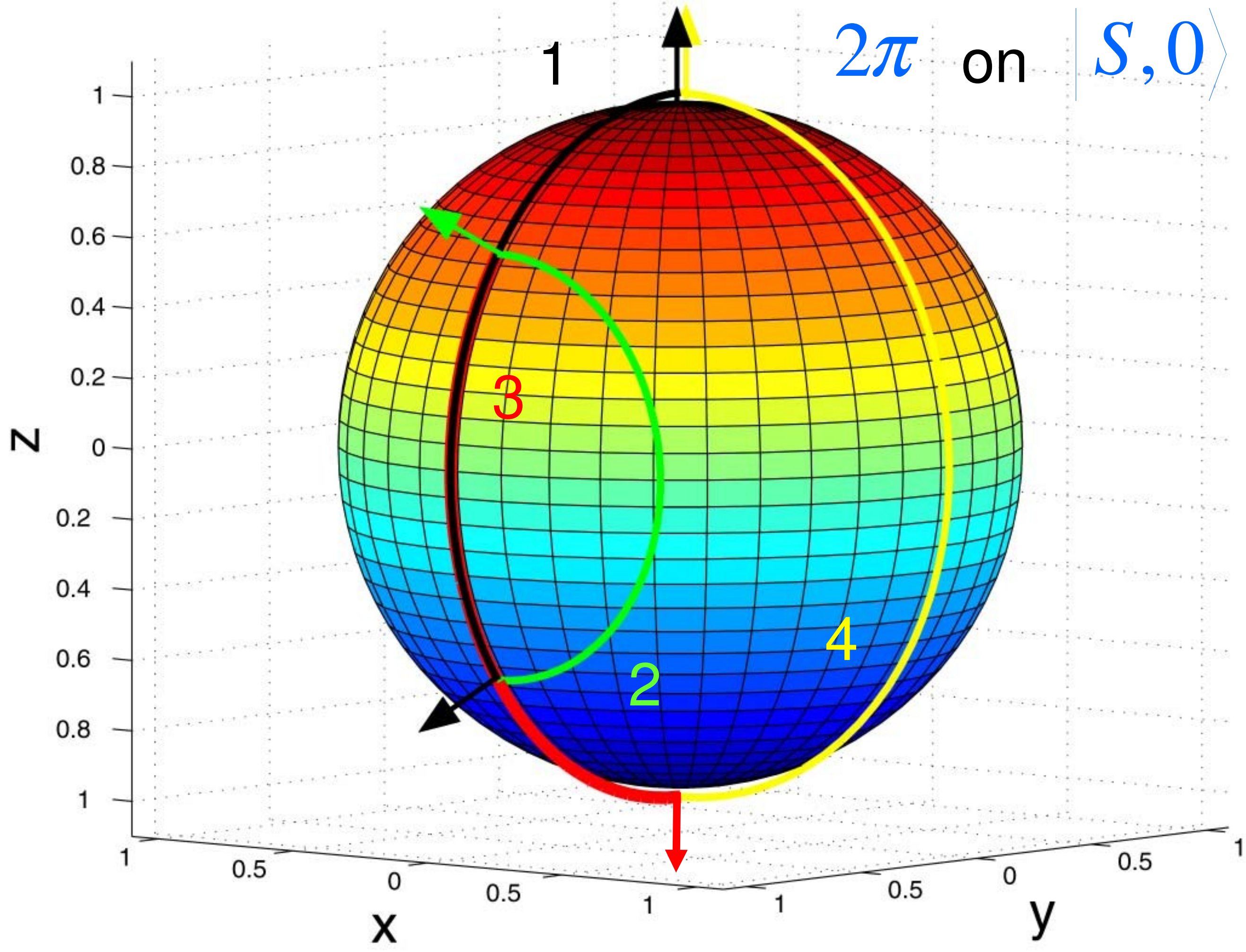
Composite 2π-rotation:



A phase gate with 4 pulses (2π rotation)

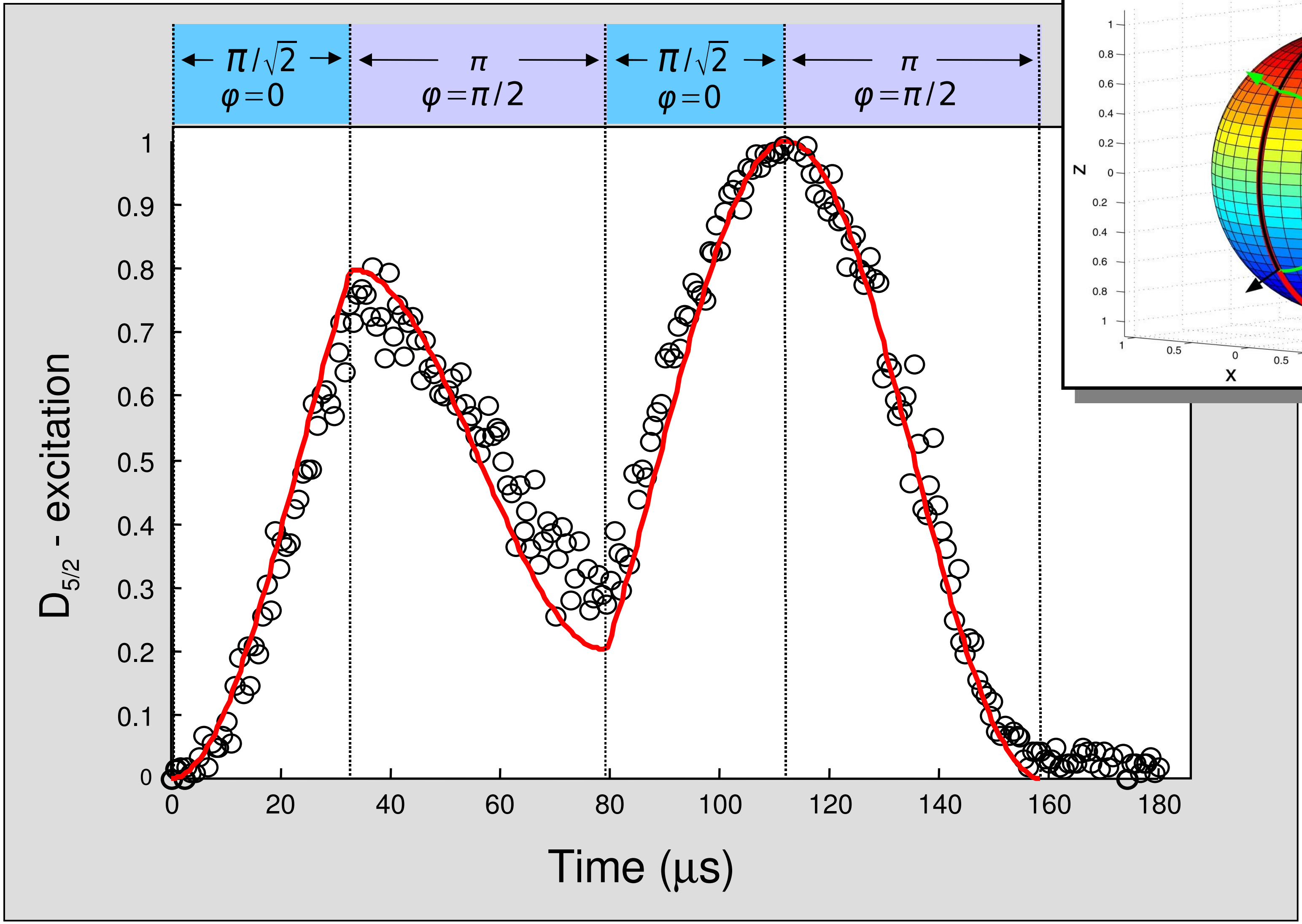
$$R(\theta,\phi) = R_1^+(\pi,\pi/2)\, R_1^+\left(\pi/\sqrt{2},0\right) R_1^+(\pi,\pi/2)\, R_1^+\left(\pi/\sqrt{2},0\right)$$

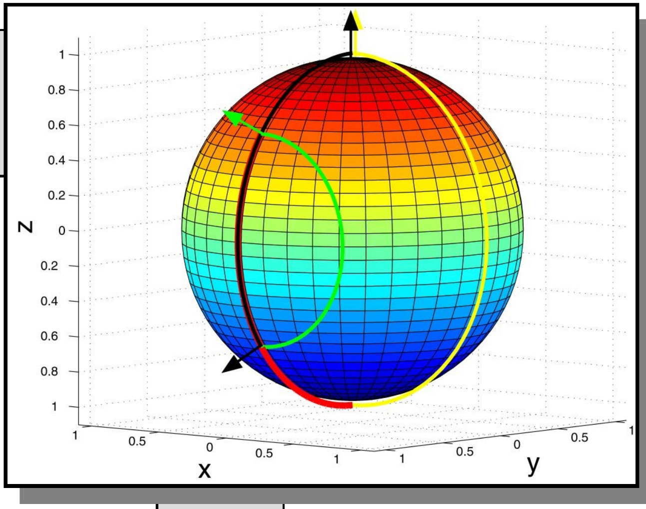
2π on $|S,0\rangle$ $|D,1\rangle$



A single ion composite phase gate: Experiment

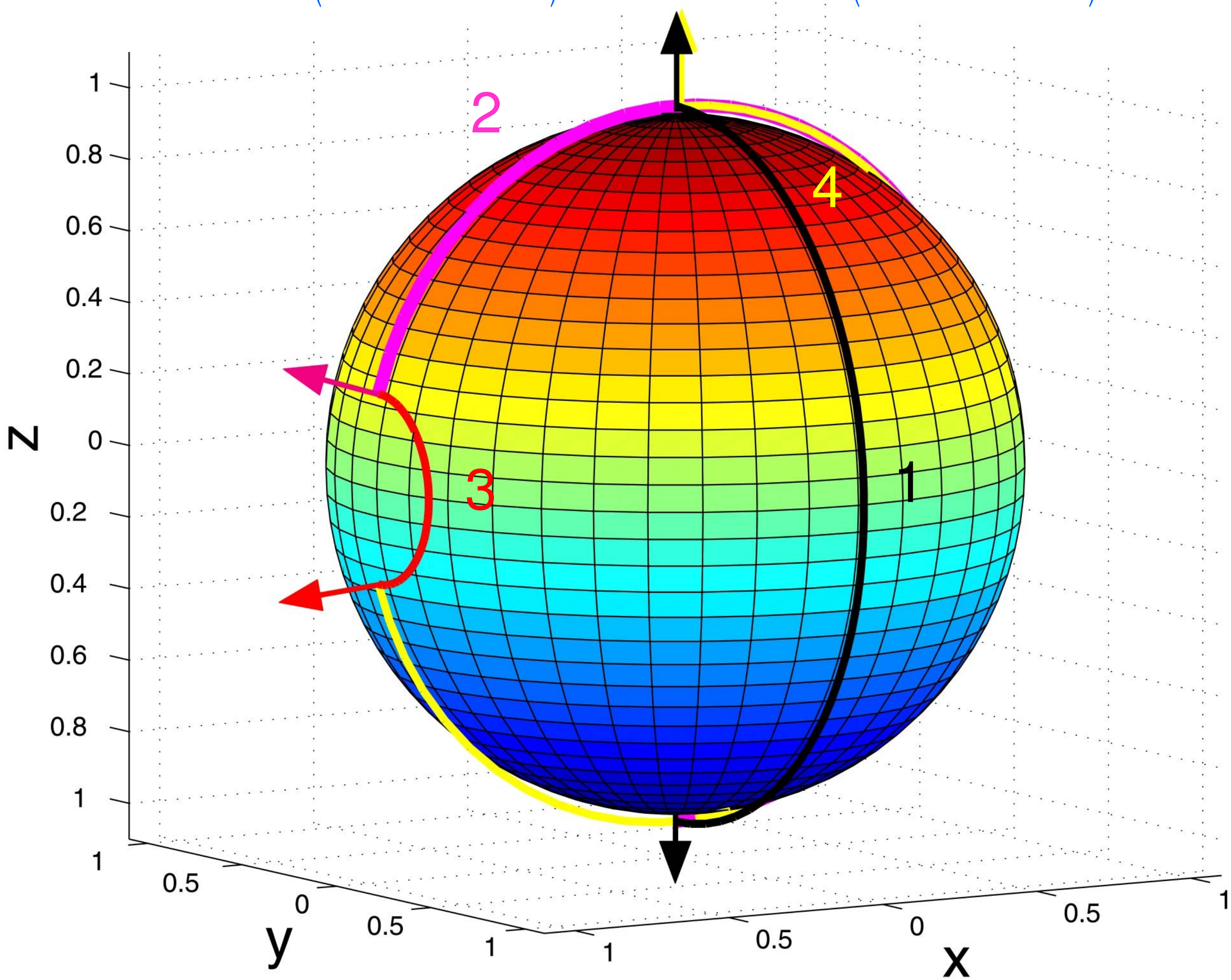
state preparation $|S,0\rangle$, then application of phase gate pulse sequence



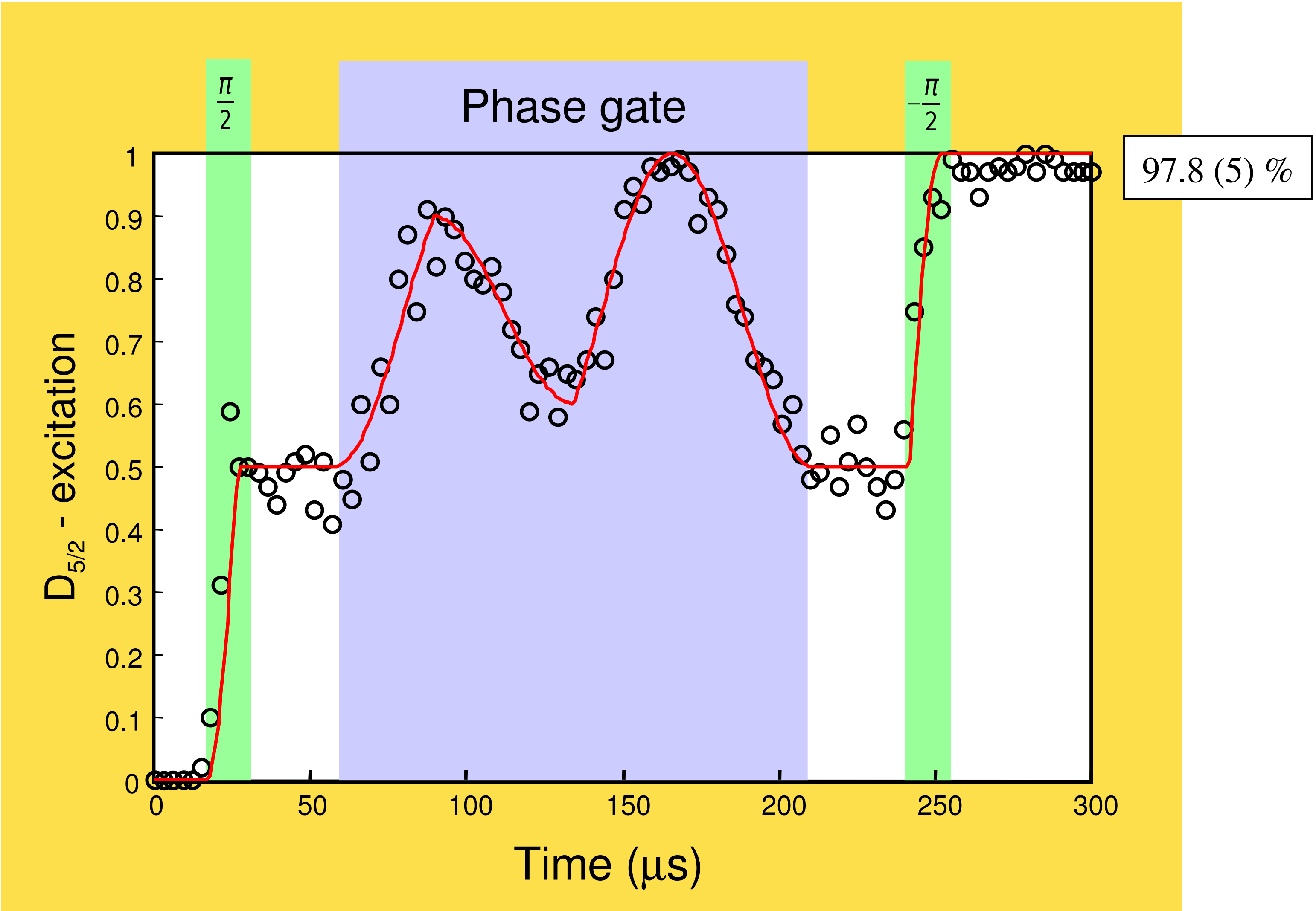


Population of |S,1> - |D,2> remains unaffected

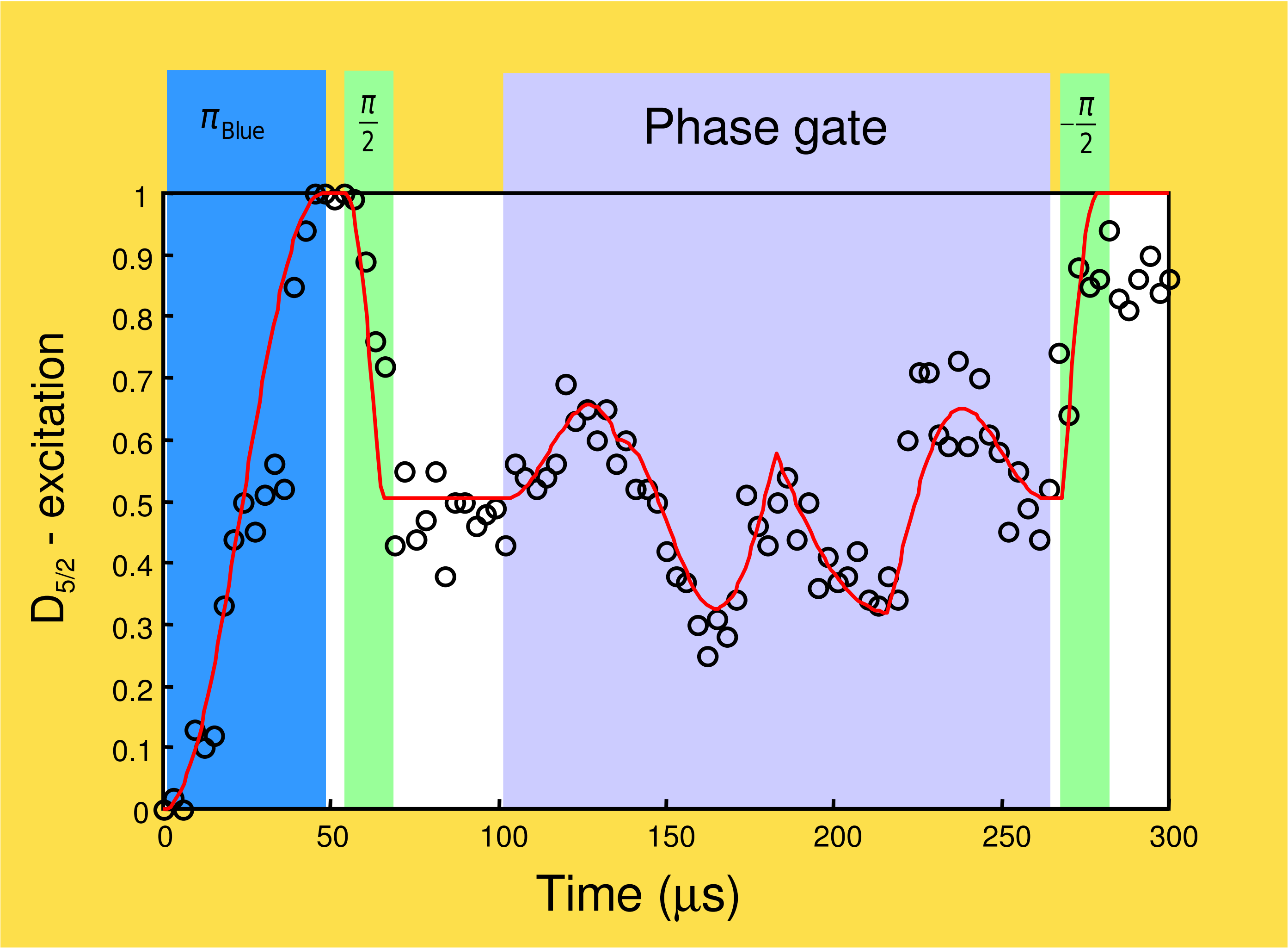
$$R(\theta,\phi) = R_1^+\left(\pi\sqrt{2},\pi/2\right) R_1^+\left(\pi,0\right) R_1^+\left(\pi\sqrt{2},\pi/2\right) R_1^+\left(\pi,0\right)$$



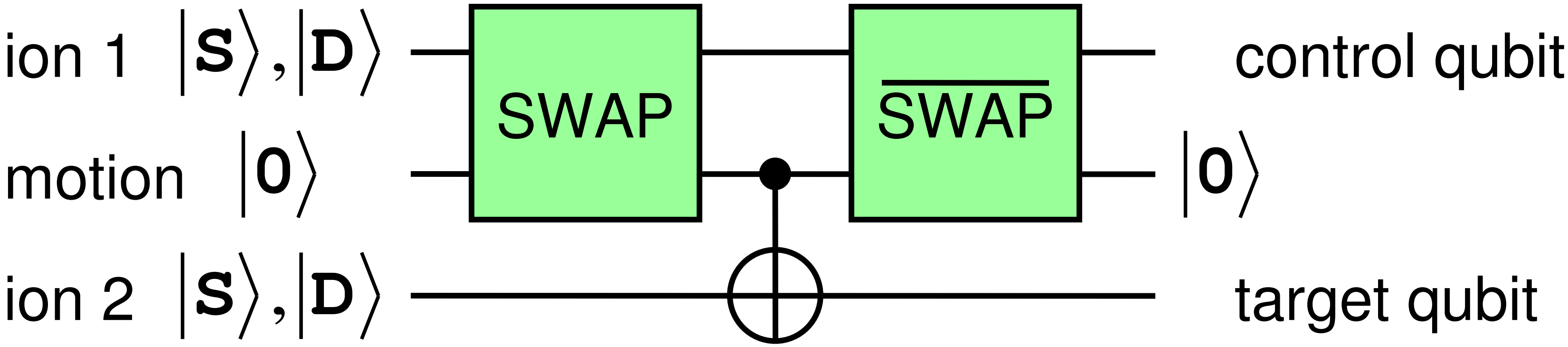
Testing the phase of the phase gate |0,S>



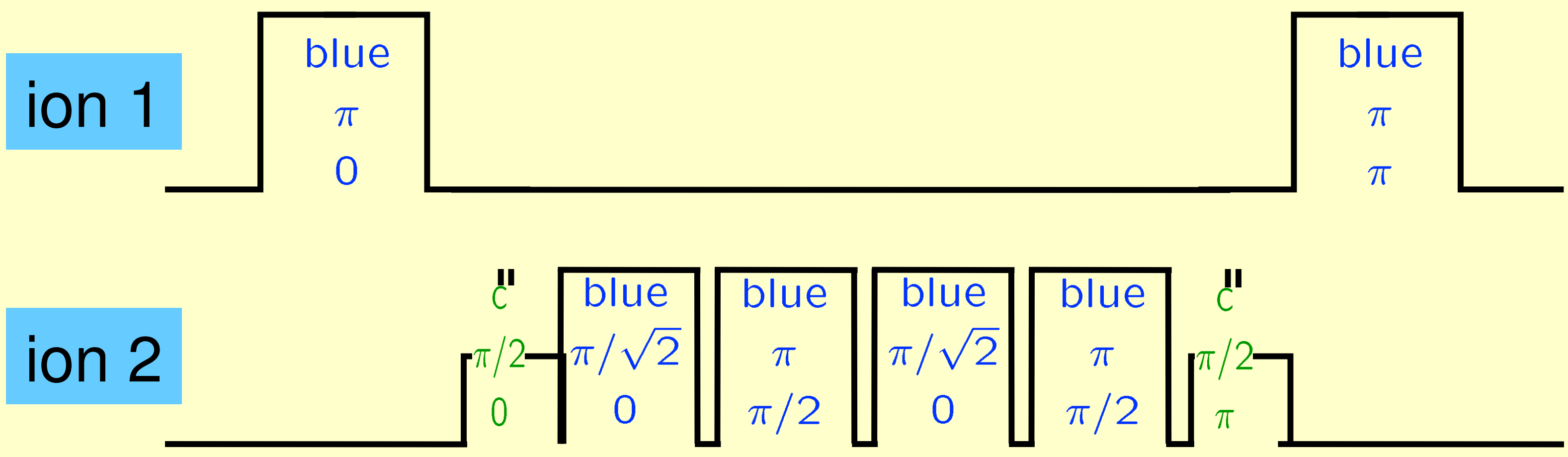
Phase gate with starting in |D,1>



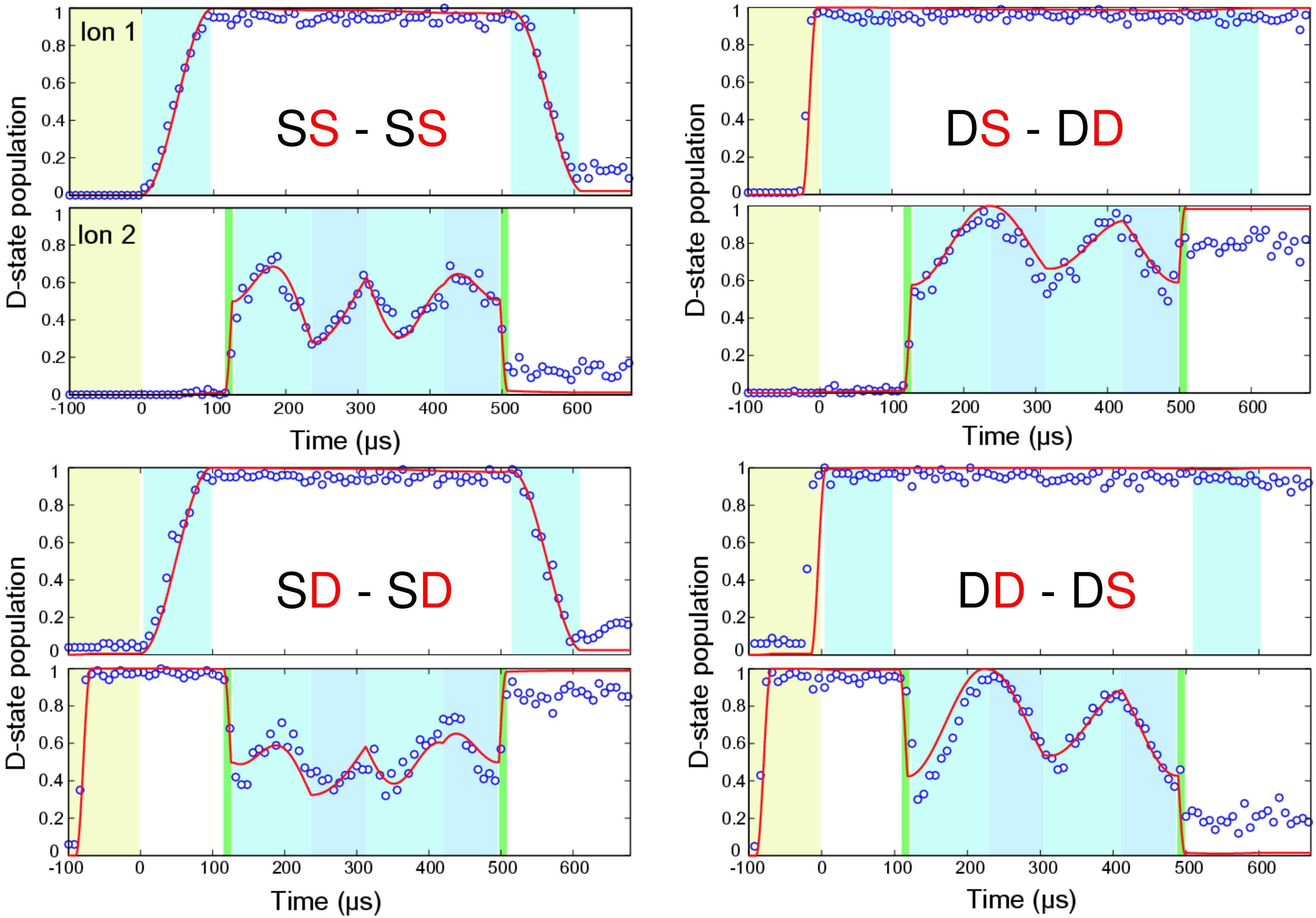
Cirac - Zoller two-ion controlled-NOT operation



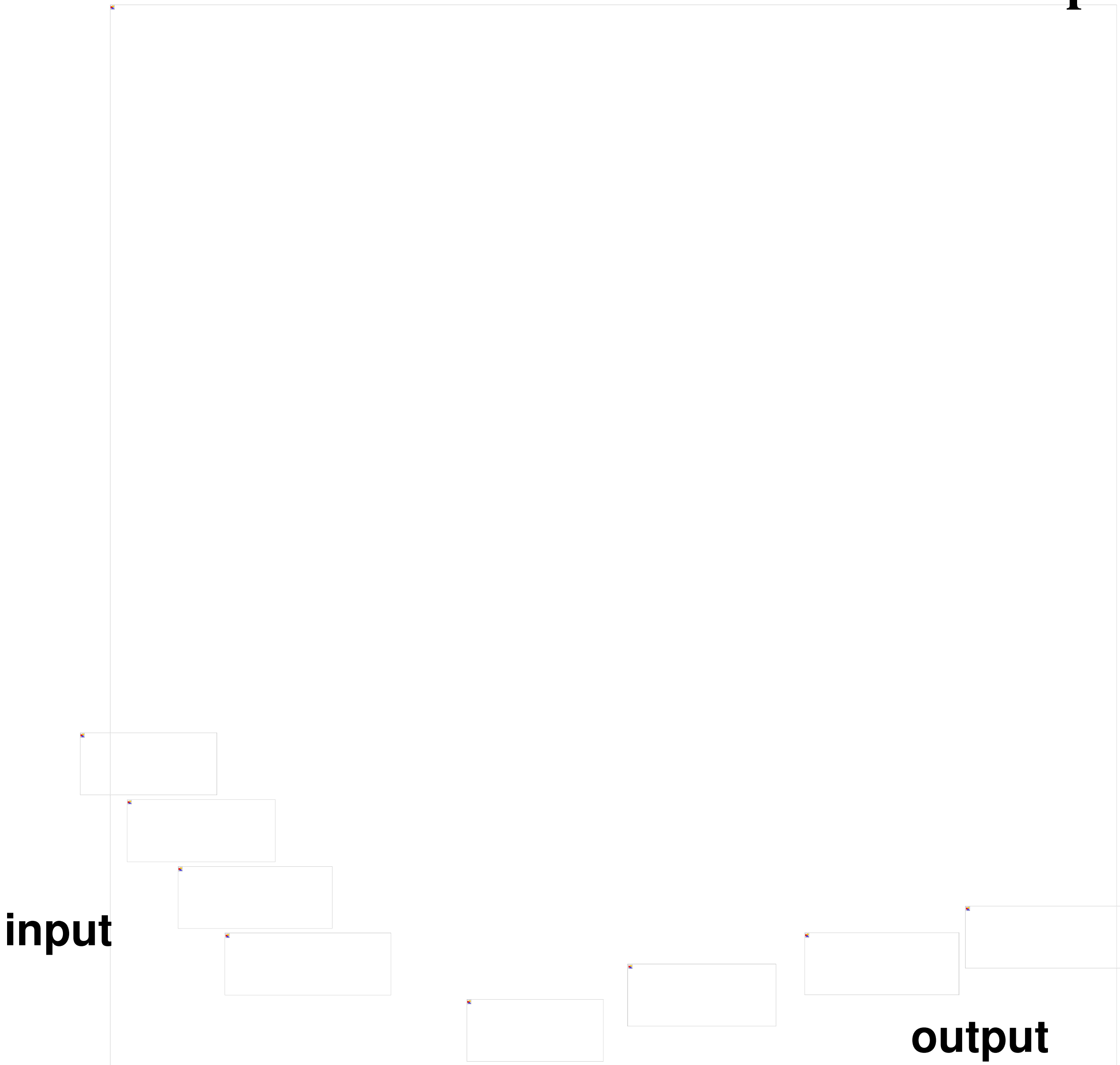
pulse sequence:



Cirac – Zoller CNOT gate operation

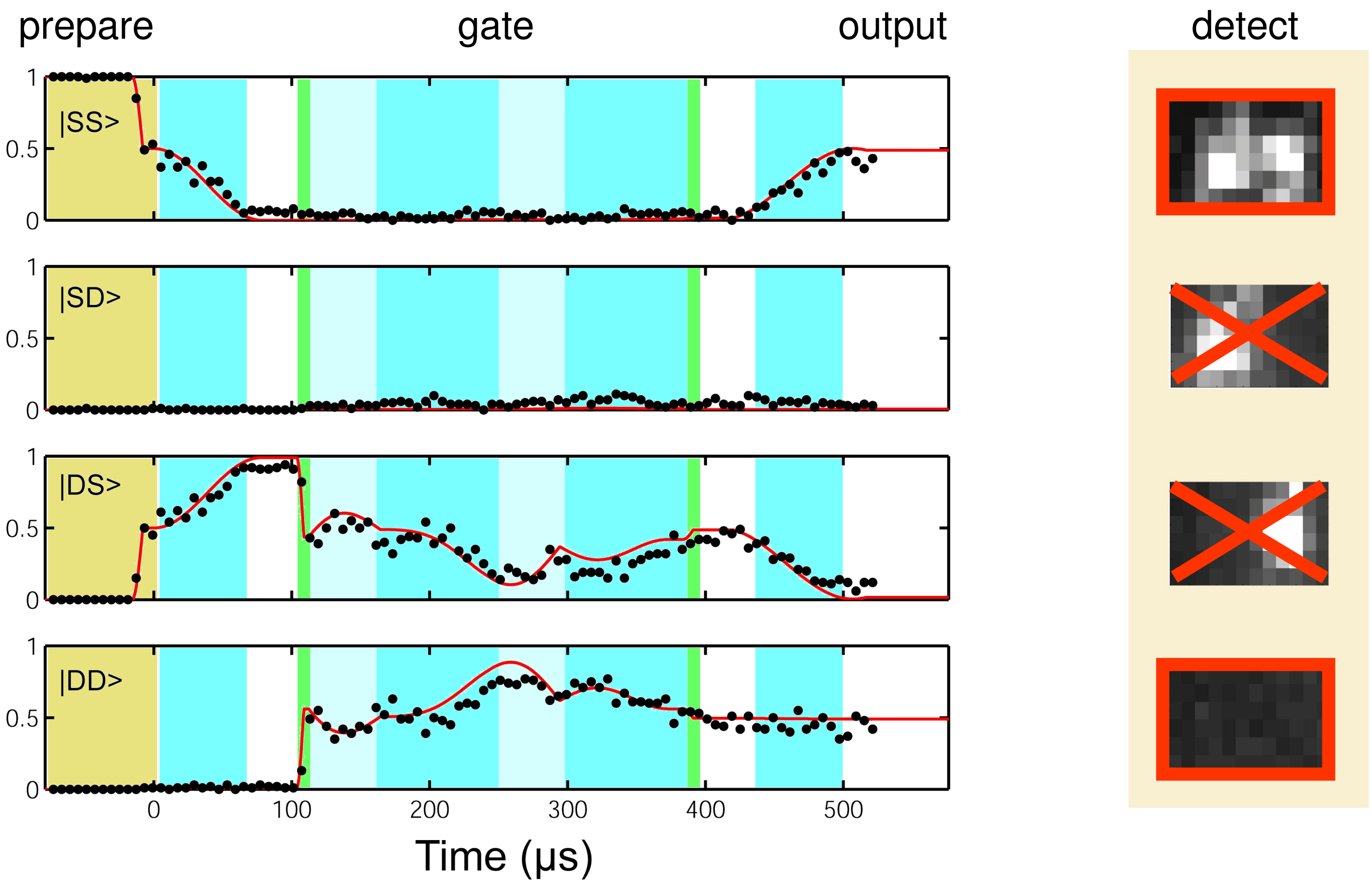


Measured truth table of Cirac-Zoller CNOT operation



Superposition as input to CNOT gate

$$|D+S\rangle|S\rangle \longrightarrow |DD\rangle + |SS\rangle$$



Error budget for Cirac-Zoller CNOT

Error source	Magnitude	Population loss
Laser frequency noise (Phase coherence)	~ 100 Hz (FWHM)	~ 10 % !!!
Residual thermal excitation	$\langle n\rangle_{bus} < 0.02$ $\langle n\rangle_{spec} = 6$	2 % 0.4 %
Laser intensity noise	1 % peak to peak	0.1 %
Addressing error (can be corrected for partially)	5 % in Rabi frequency (at neighbouring ion)	3 %
Off resonant excitations	for $t_{gate} = 600\ \mu s$	4 %
Laser detuning error	~ 500 Hz (FWHM)	~ 2 %
Total	November 2002	~ 20 %

Error budget for Cirac-Zoller CNOT

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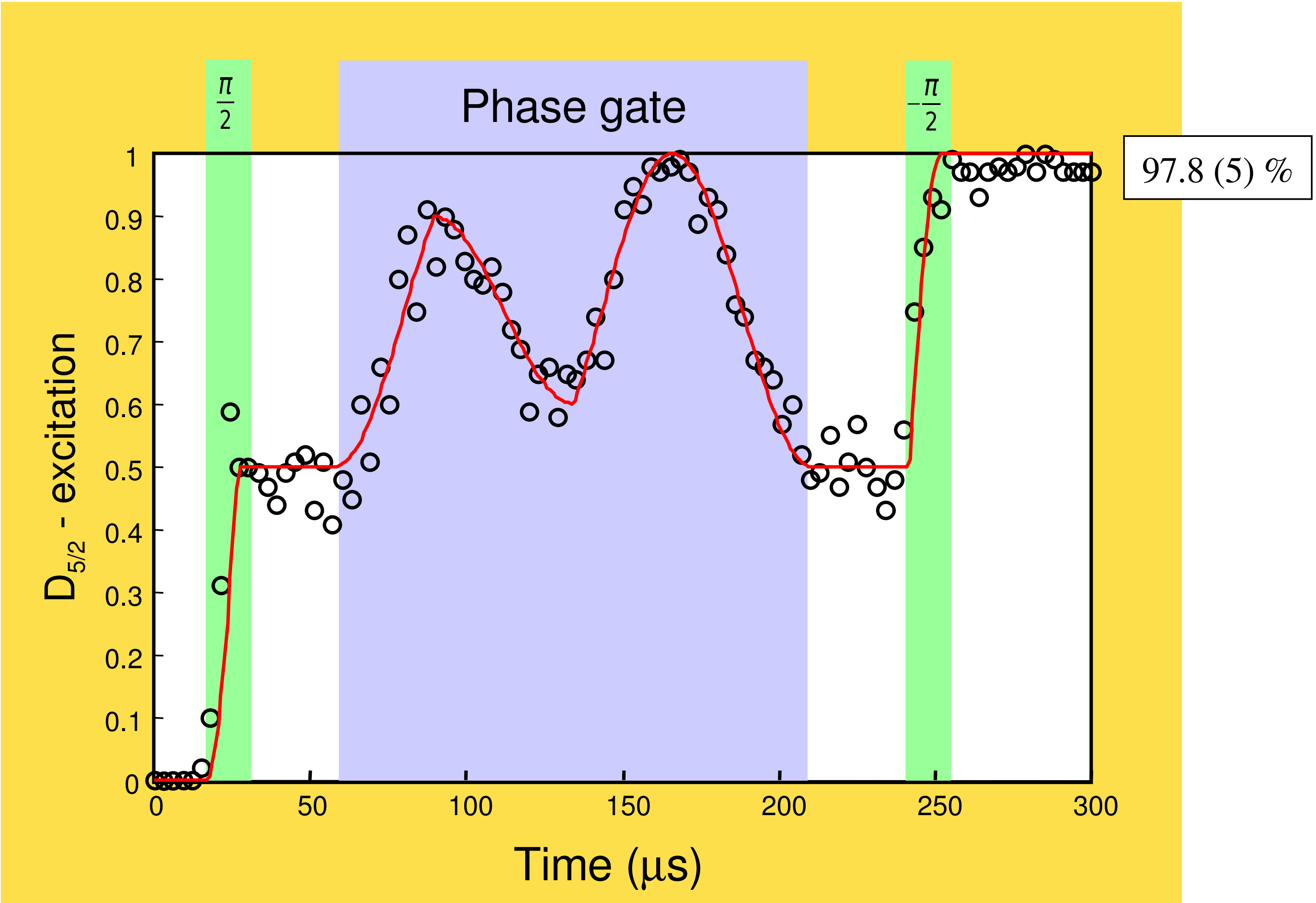
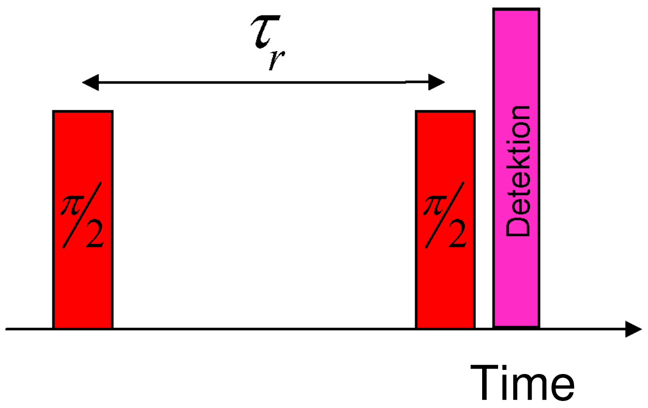
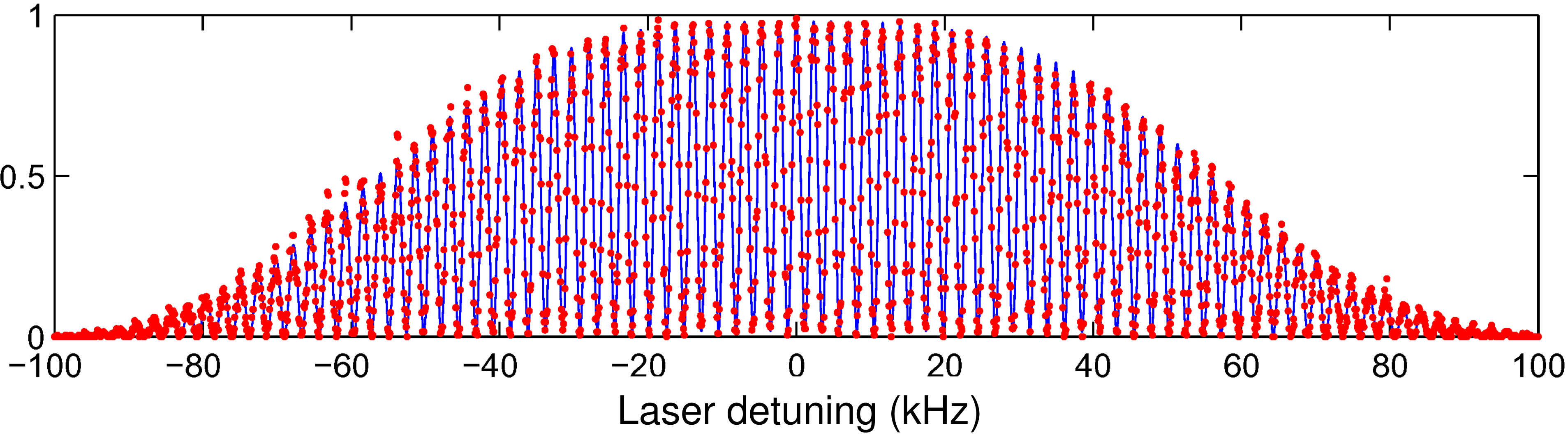
Testing the phase of the phase gate |0,S>



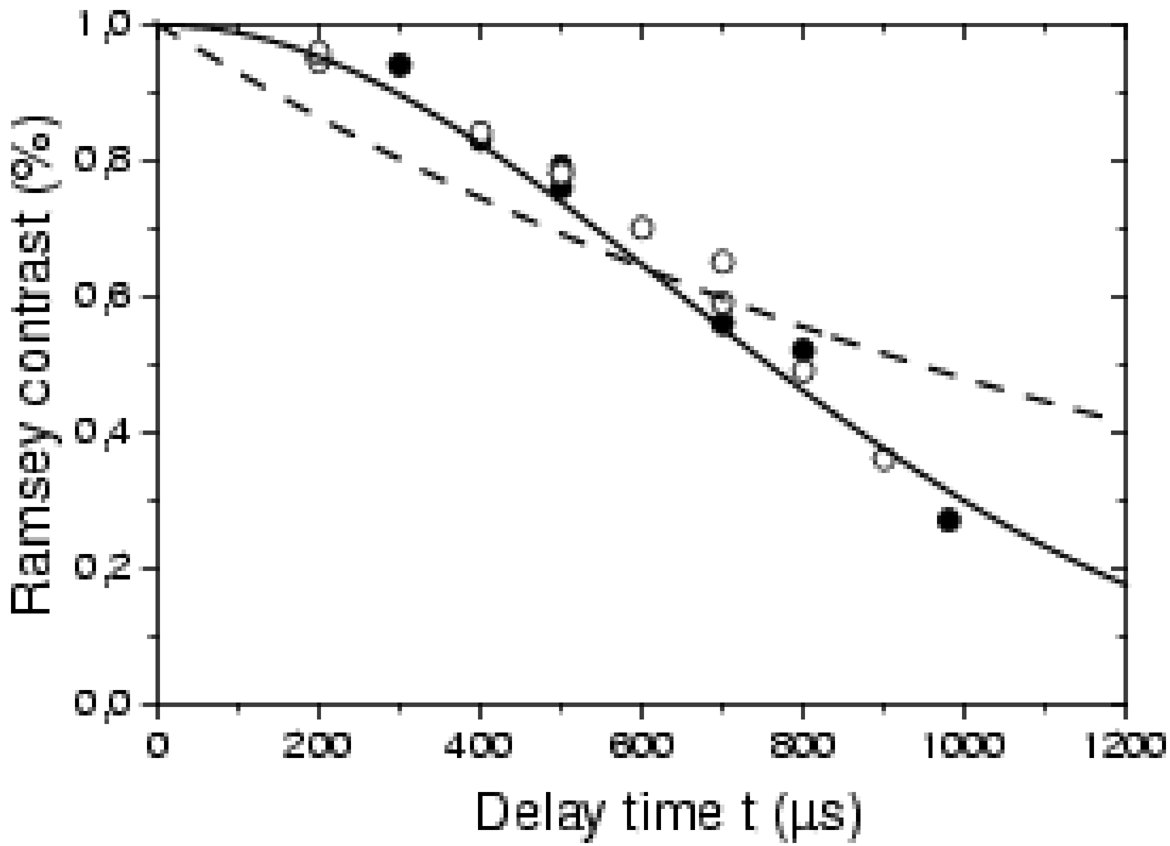
Ramsey experiment



Quantum algorithms can be viewed as generalized Ramsey experiments!

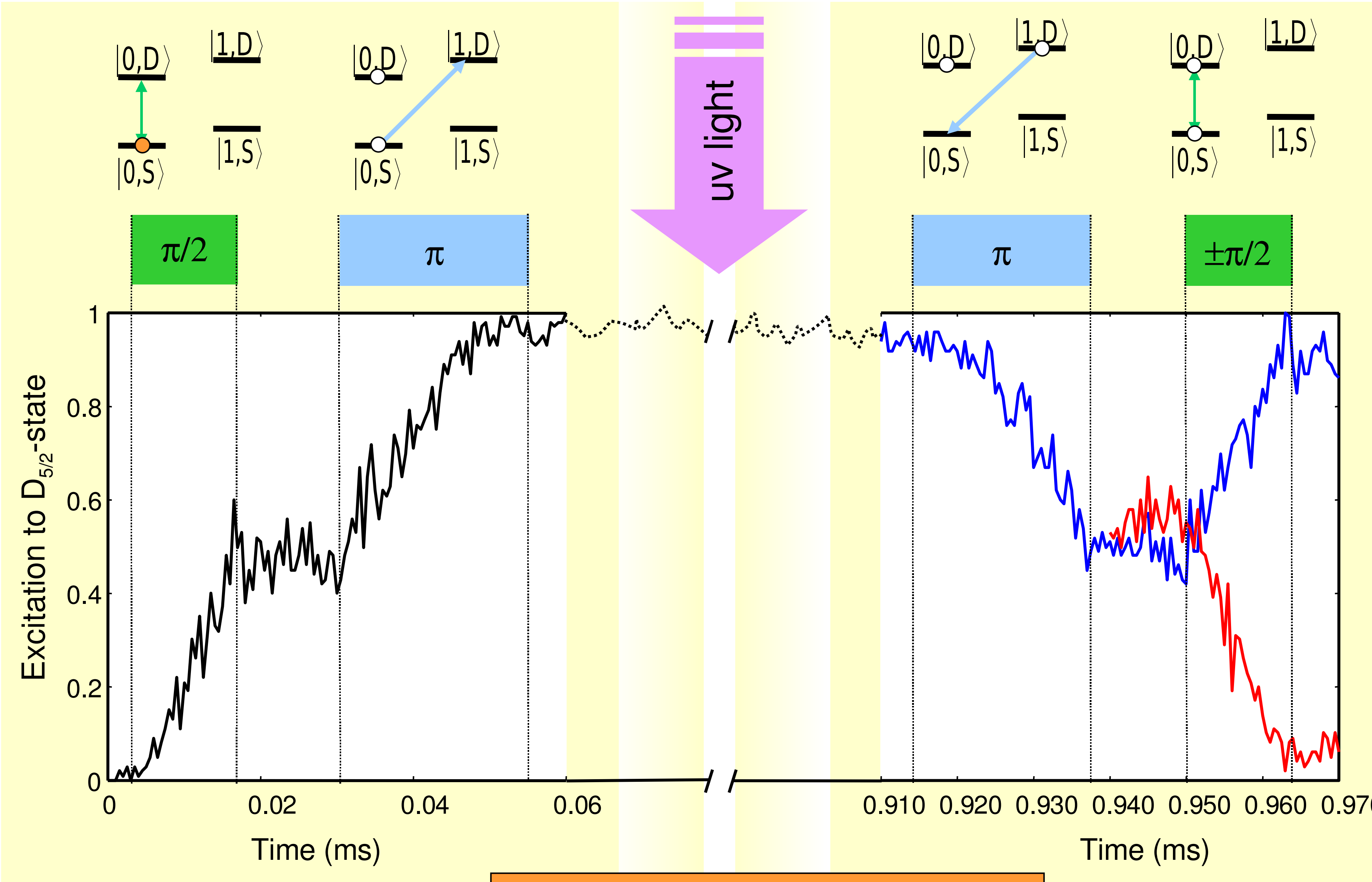


Phase coherence



⇒ Gaussian modell yields a coherence time of 0.9 ms
Now (in 2005) 2 ms are more typical.

Test of the motional coherence



After 60 ms still 50 % contrast!

Error budget for Cirac-Zoller CNOT

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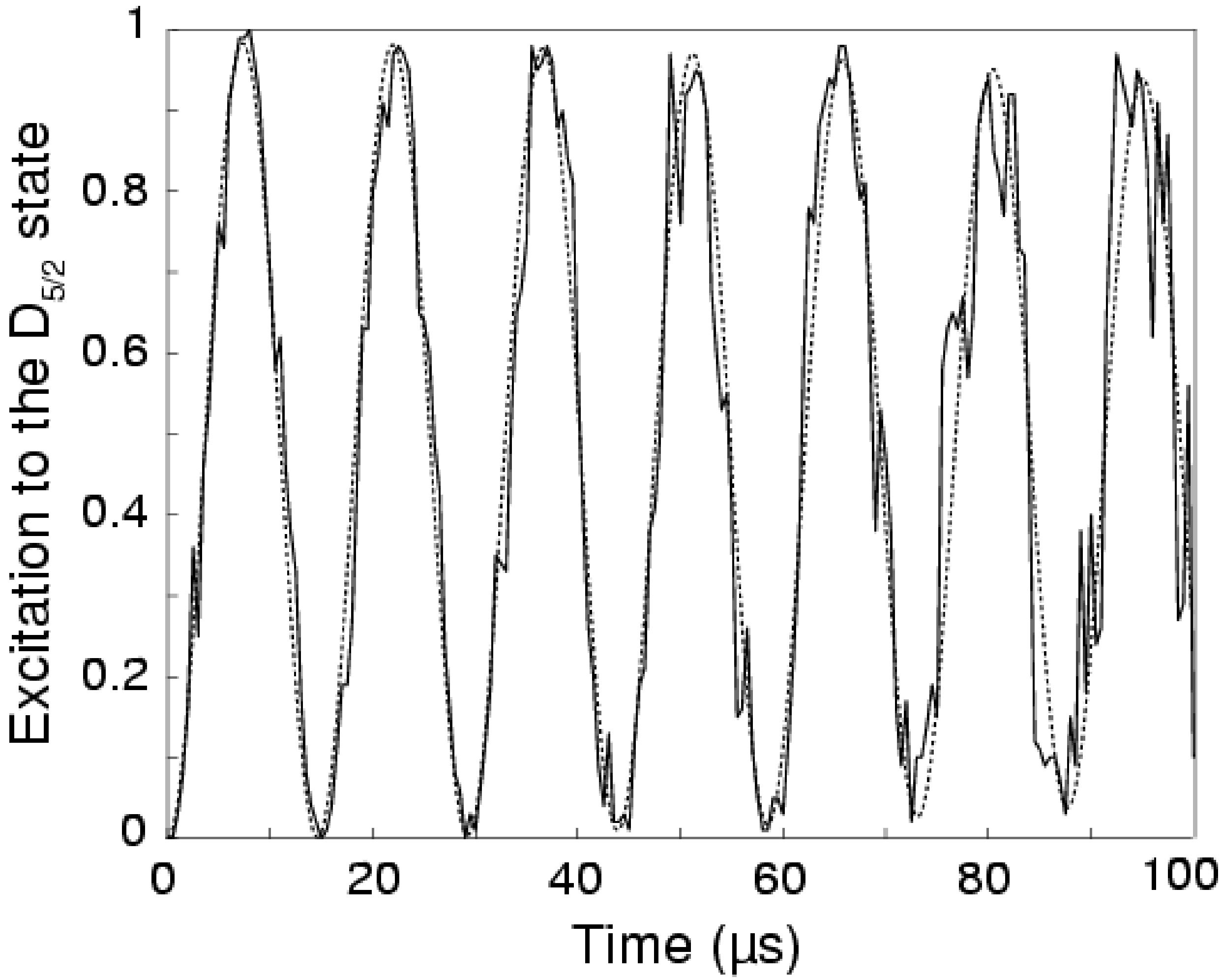
Intensity noise



=> Laser intensity noise: 0.03

Error budget for Cirac-Zoller CNOT

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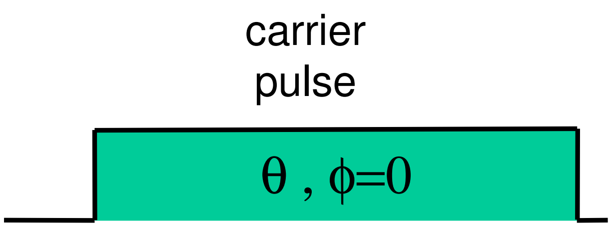
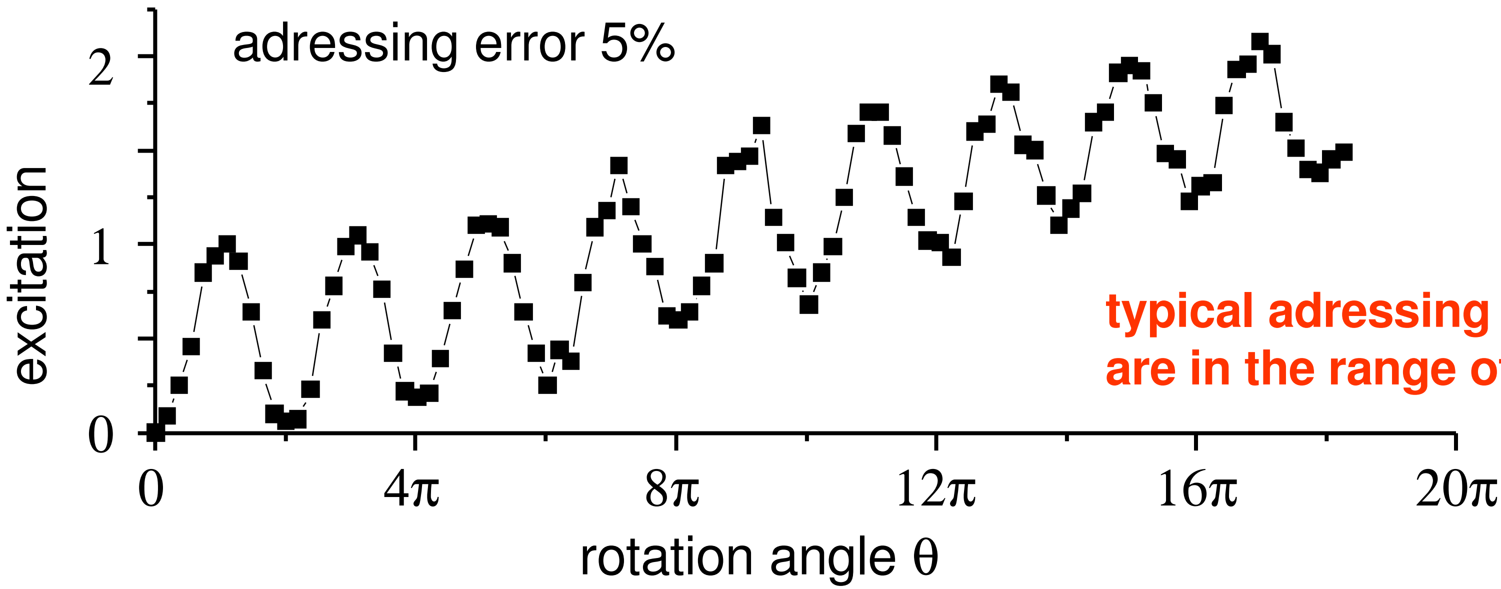
Qubit-Rotations with two ions

Pulse sequence:



$$\textbf{error} = \frac{\Omega_{\text{neighbour}}}{\Omega_{\text{target}}} = \sqrt{\frac{I_{\text{neighbour}}}{I_{\text{target}}}}$$



typical adressing errors are in the range of 3-5%

Error budget for Cirac-Zoller CNOT

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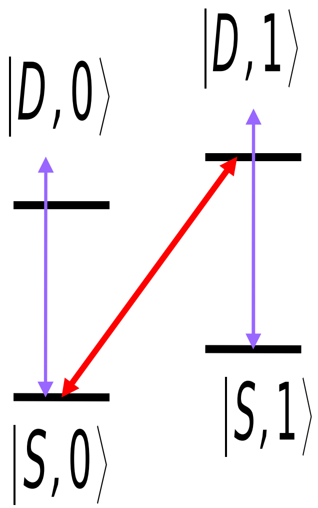
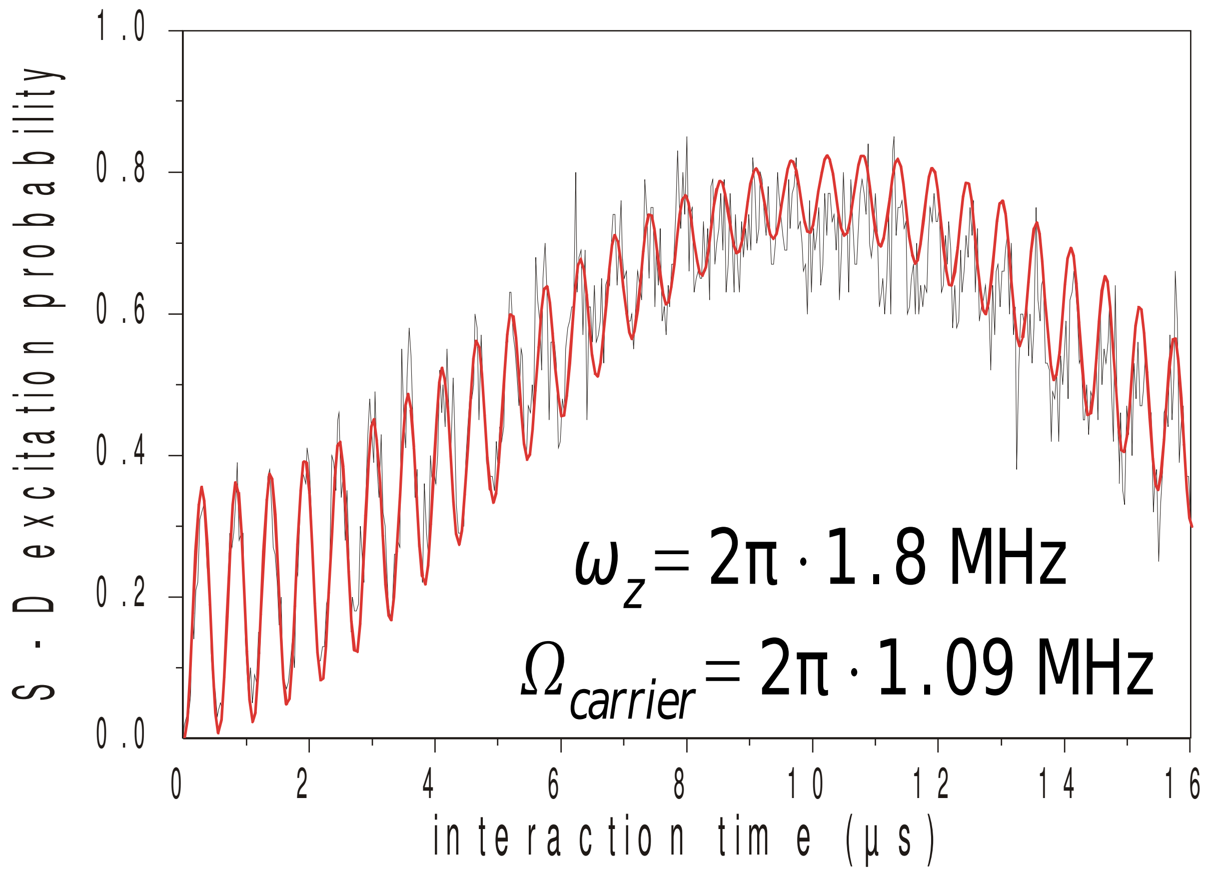
Off-resonant carrier excitation





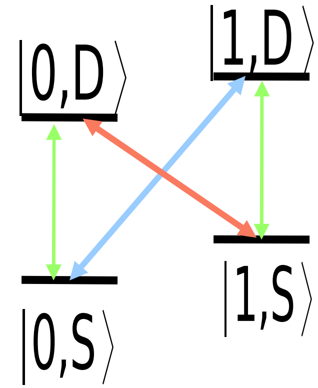
Problem:

- AC Stark shifts
- off-resonant (carrier) excitation (spectator modes)

A. Steane, C. F. Roos, D. Stevens, A. Mundt, D. Leibfried, F. Schmidt-Kaler, R. Blatt, Phys. Rev. A **62**, 042305 (2000)

Computation method

• Solve master equation!



Schrödinger equation for 1 ion:

$$
\begin{array}{cc}
 & \begin{array}{cccc} |0,S\rangle & |0,D\rangle & |1,S\rangle & |1,D\rangle \end{array} \\
i\frac{d}{dt}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = & \begin{pmatrix} 0 & \Omega_0 & 0 & -i\eta\,\Omega_0 \\ \Omega_0 & \Delta & i\eta\,\Omega_0 & 0 \\ 0 & -i\eta\,\Omega_0 & \omega_T & (1-\eta^2)\,\Omega_0 \\ i\eta\,\Omega_0 & 0 & (1-\eta^2)\,\Omega_0 & \Delta+\omega_T \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} \begin{array}{c} |0,S\rangle \\ |0,D\rangle \\ |1,S\rangle \\ |1,D\rangle \end{array}
\end{array}
$$

$\eta \approx 0.03$: Lamb-Dicke-parameter / n: Number of phonons

Δ: Laser detuning / ω_T : Trap frequency

Hilbert space: $|0,1,2\rangle \otimes |S,D,D_{aux}\rangle_1 \otimes |S,D,D_{aux}\rangle_2$ (27 dimensions)

Error budget for Cirac-Zoller CNOT

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Total	November 2002	~ 20 %

Summary of the complications of the Cirac-Zoller approach

- **the gate is slow (off-resonant excitations)**
  => sensitive to frequency fluctuations / deviations.

- **the gate requires addressing** => trap frequency needs to reduced
  Use other gate types which do not require addressing.

- **the gate is sensitive to motional heating**
  Not really a problem yet.

Scaling of this approach?

Problems :

- Coupling strength between internal and motional states of a N-ion string decreases as

$$\eta \propto \frac{1}{\sqrt{N}}$$

(momentum transfer from photon to ion string becomes more difficult)

-> Gate operation speed slows down

- More vibrational modes increase risk of spurious excitation of unwanted modes

- Distance between neighbouring ions decreases -> addressing more difficult

-> Use flexible trap potentials to split long ion string into smaller segments and perform operations on these smaller strings

AG Quantenoptik und Spektroskopie

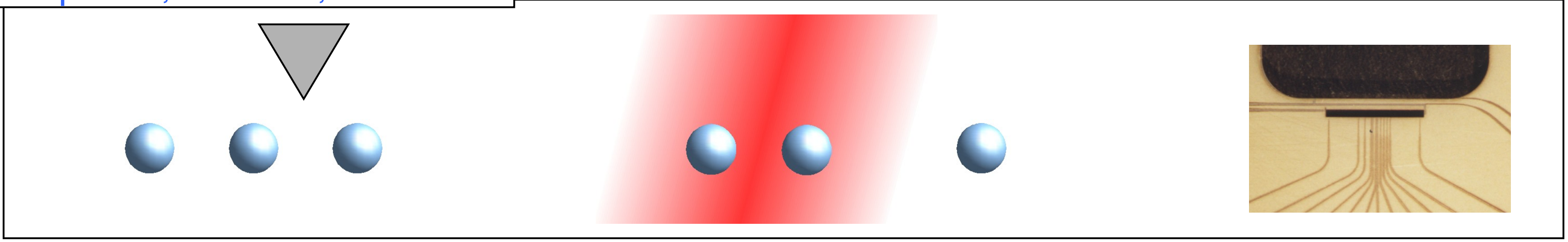
Scaling ion trap quantum computers

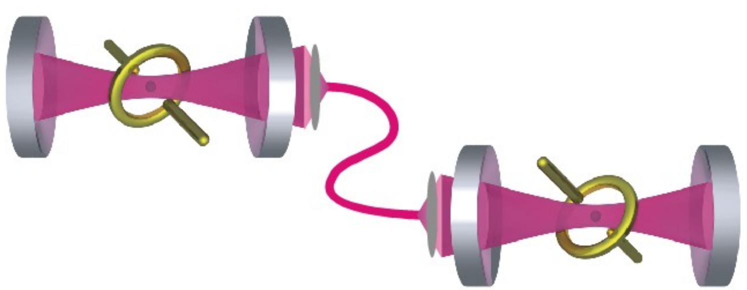
Easy to have thousands of ions in a trap and to manipulate them individually…

but it is hard to control their interaction!

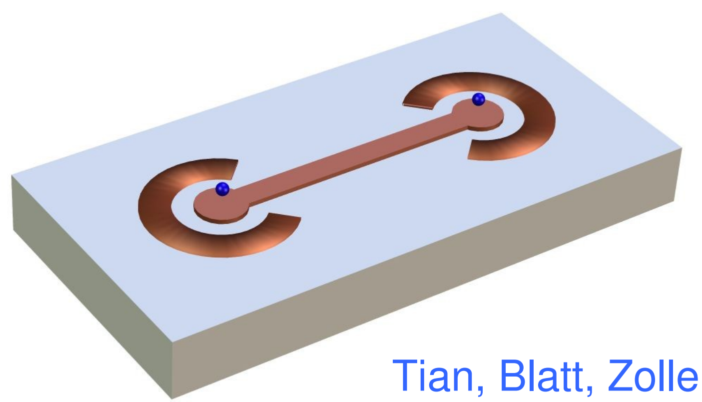
The solutions:

Kielpinski, Monroe, Wineland





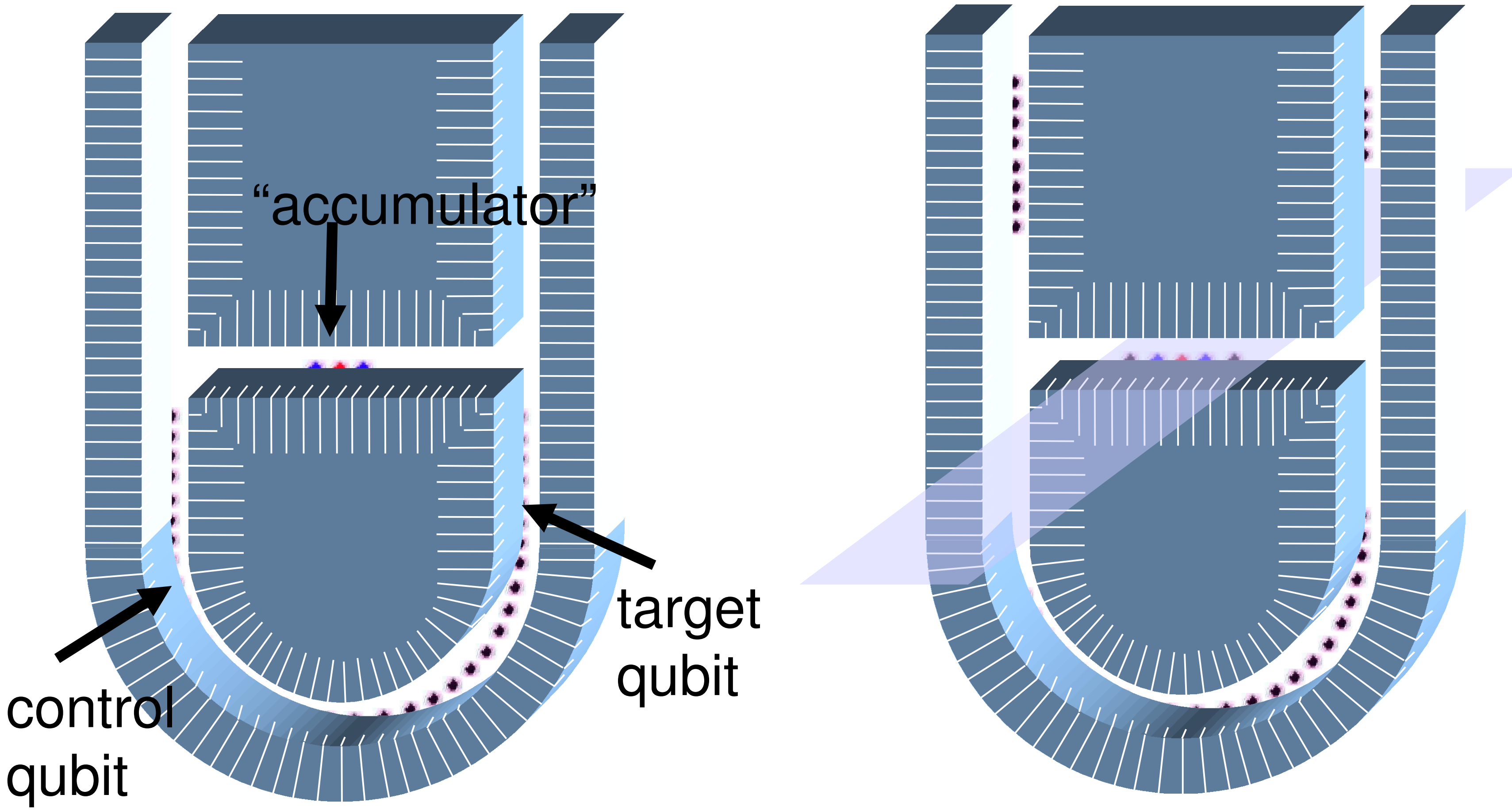
Cirac, Zoller, Kimble, Mabuchi



Tian, Blatt, Zoller

Idea #1: move the ions around

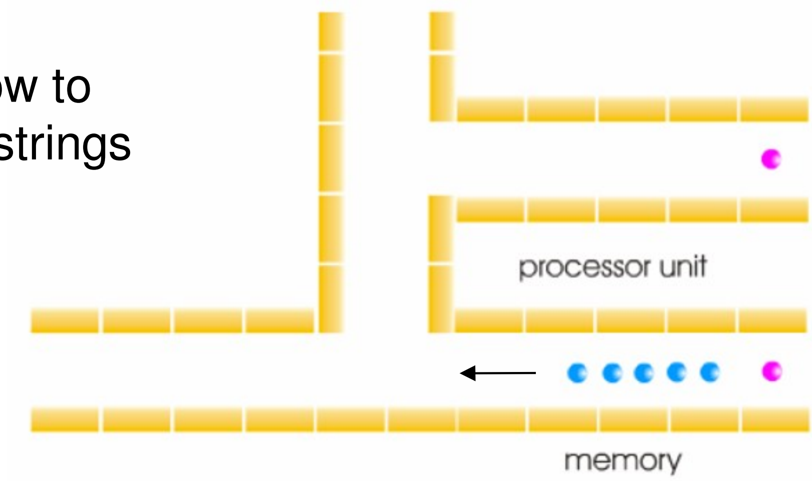
D. Wineland, Boulder, USA
Kieplinski et al, Nature 417, 709 (2002)



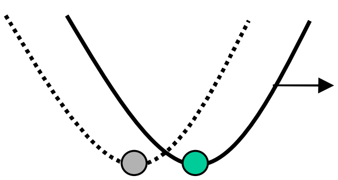
Segmented ion traps as scalable trap architecture

(ideas pioneered by D. Wineland, NIST, and C. Monroe, Univ. Michigan)

Segmented trap electrode allow to transport ions and to split ion strings



State of the art:



Transport of ions

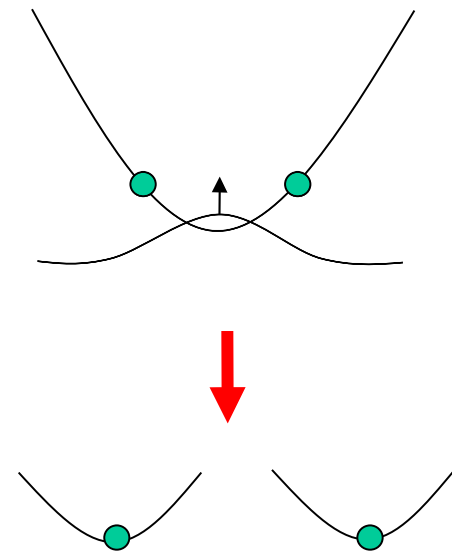
1 mm within 50 μs

no motional heating

Splitting of two-ion crystal

$t_{separation} \approx 200\ \mu s$

small heating $\Delta n \approx 1$



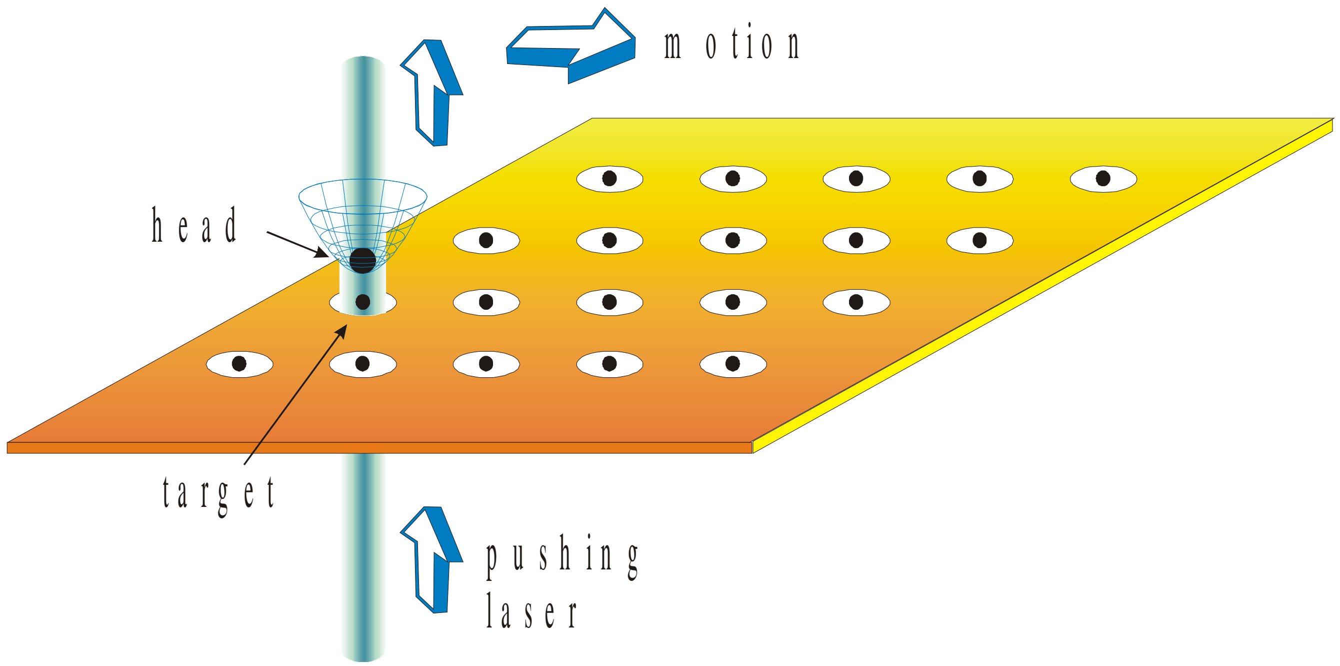
„Architecture for a large-scale ion-trap quantum computer", D. Kielpinski et al, Nature **417**, 709 (2002)

„Transport of quantum states", M. Rowe et al, quant-ph/0205084

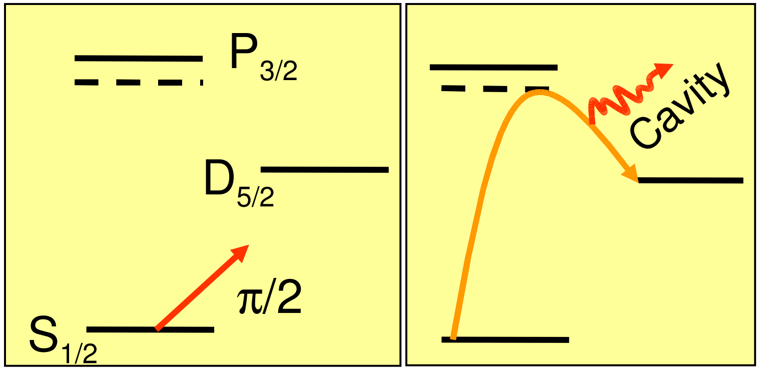
or use a head ion

I. Cirac und P. Zoller, Nature 404, 579 (2000)

- quantum optics and nano-technology: scalability



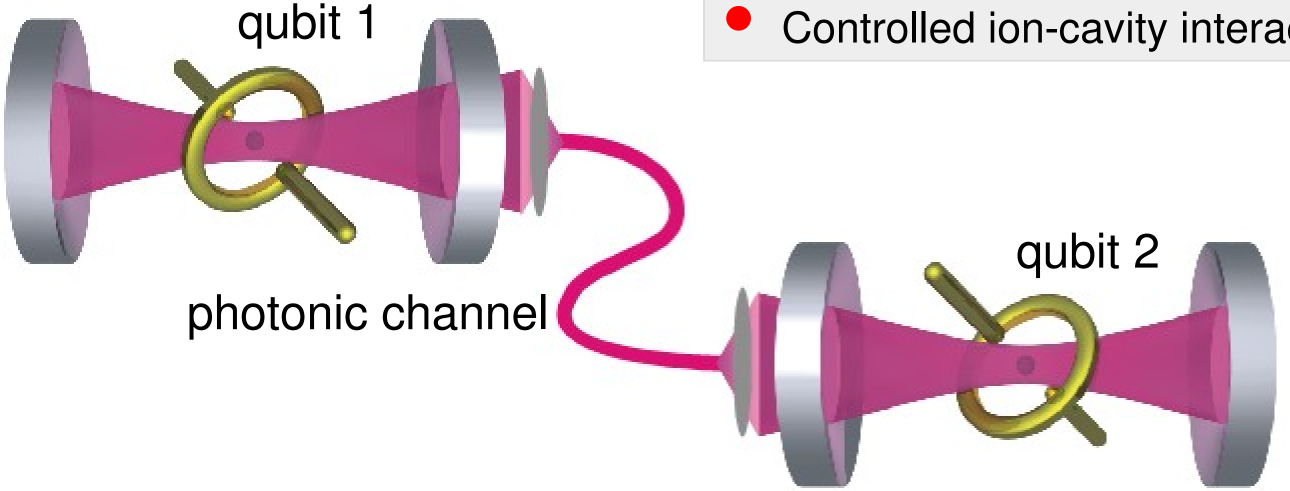
Idea #2: coupling via photons



Transfer quantum state of the ion

to cavity photon: qubit interface

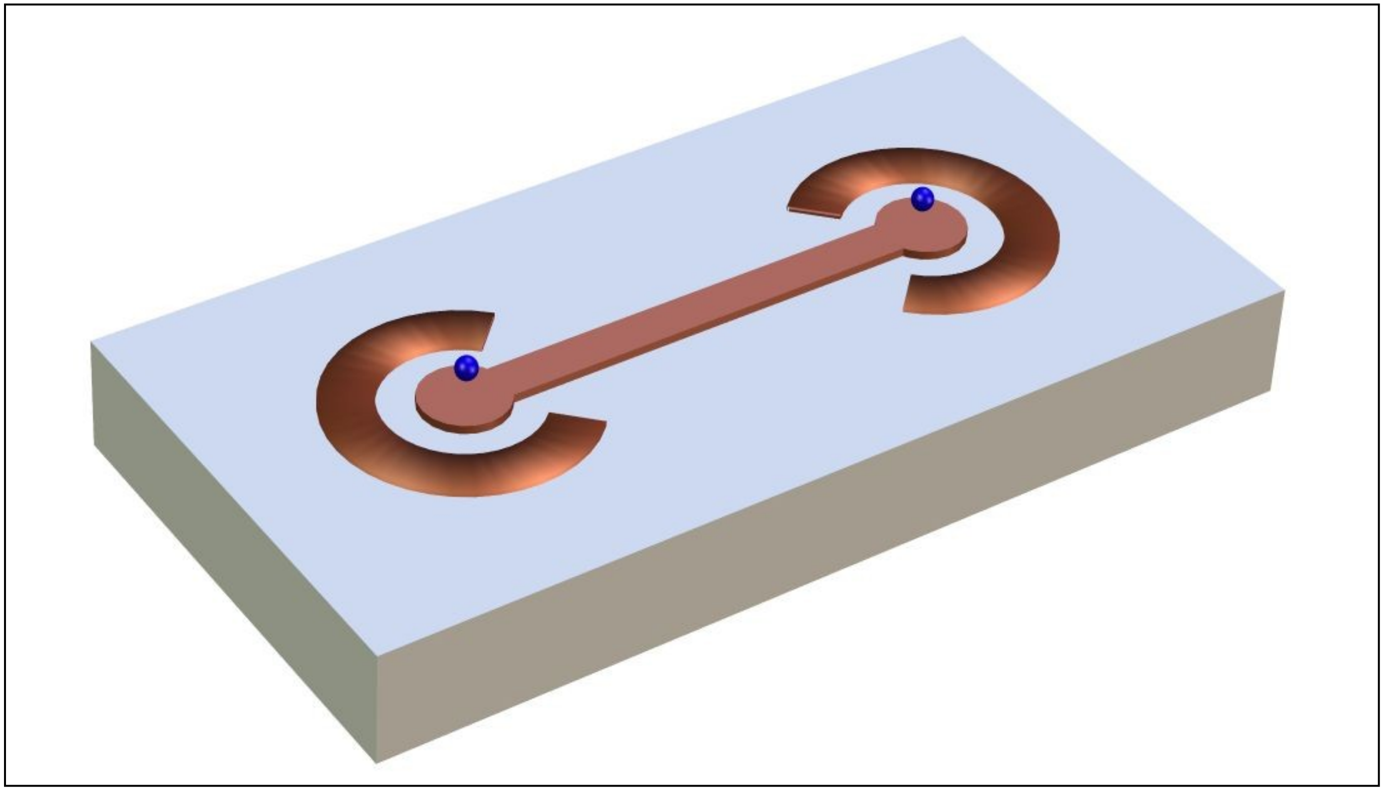
- Create superposition photon state (STIRAP)
- Detect cavity output and ion state
- Choose coupling *g* by STIRAP detuning
- Controlled ion-cavity interaction



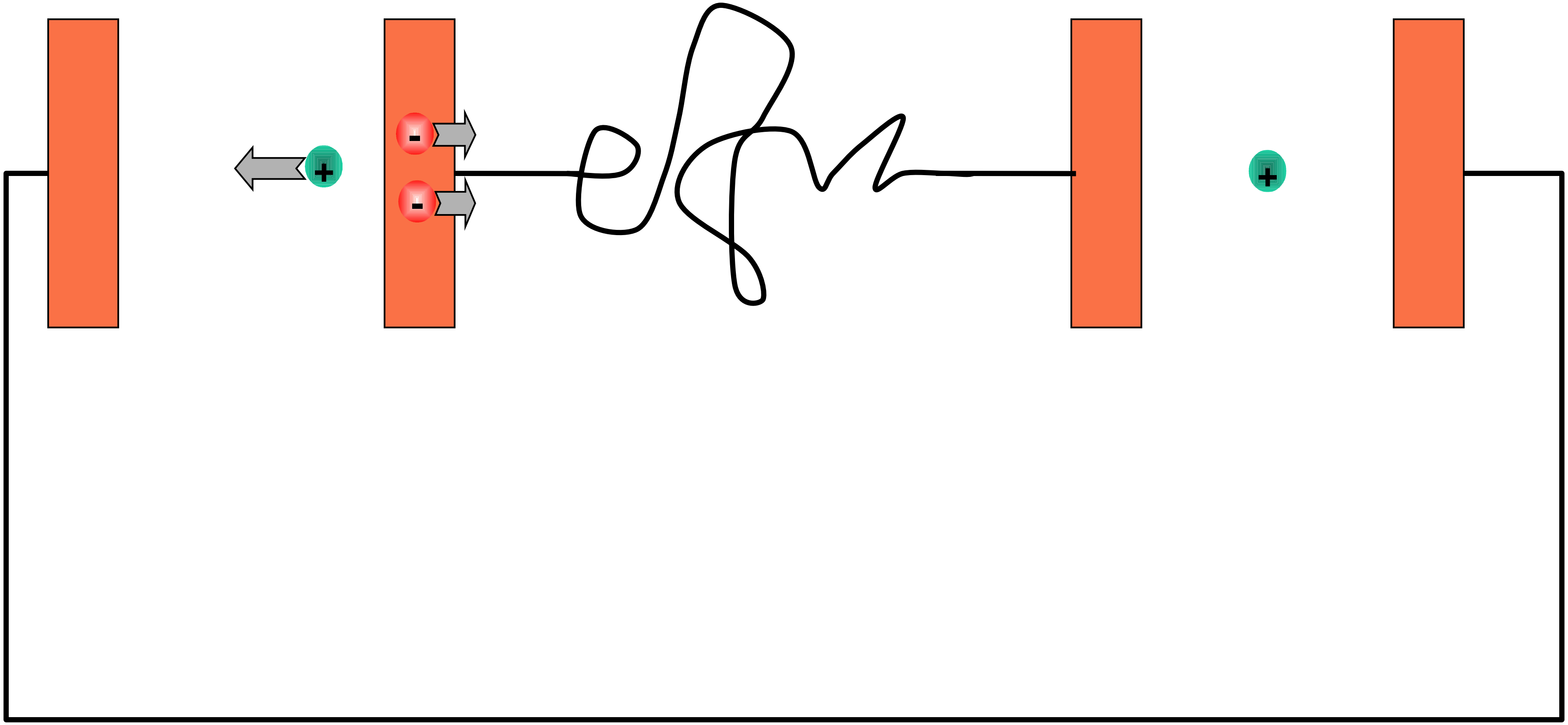
J. I. Cirac et al., PRL 78, 3221 (1997)

A. Kuhn et al., PRL 89, 067901 (2002)

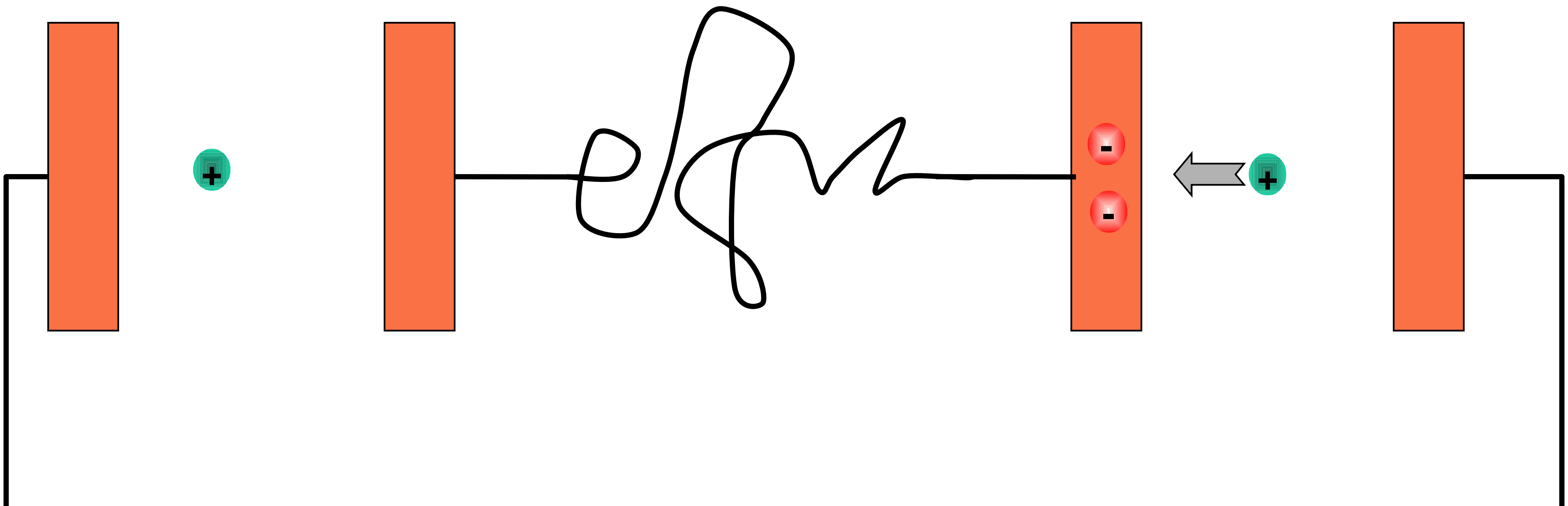
Idea #3: coupling via image charges



Idea #3: coupling via image charges



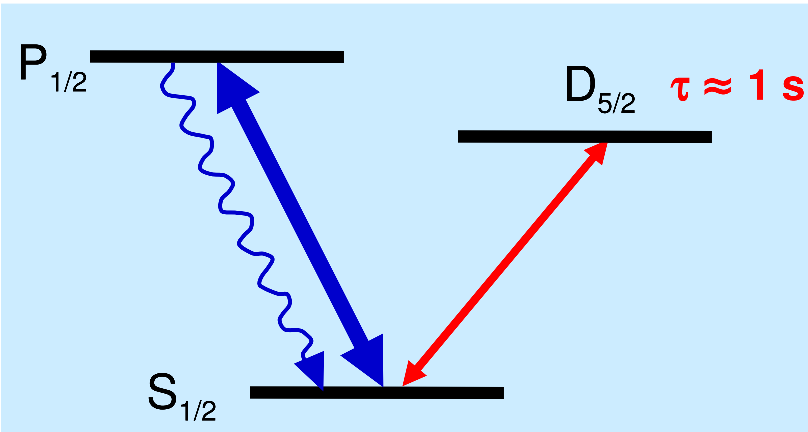
Idea #3: coupling via image charges



- Coupling in the kHz-range requires small traps (~20 μm)
- Very small currents can be measured
  => non-invasive probe for conductors (superconductors)
- Connect to solid-state qubits

Quantum information processing: Which ion is best ?

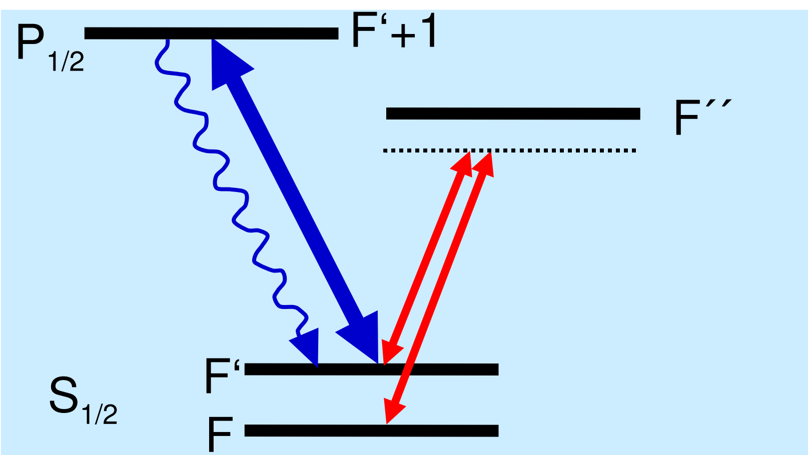
Ions with optical transition to metastable level: $^{40}Ca^+$, $^{88}Sr^+$, $^{172}Yb^+$



Qubit levels: $S_{1/2}$, $D_{5/2}$

Qubit transition: Quadrupole transition $S_{1/2} – D_{5/2}$

Ions with hyperfine structure: $^{9}Be^+$, $^{43}Ca^+$, $^{111}Cd^+$,…

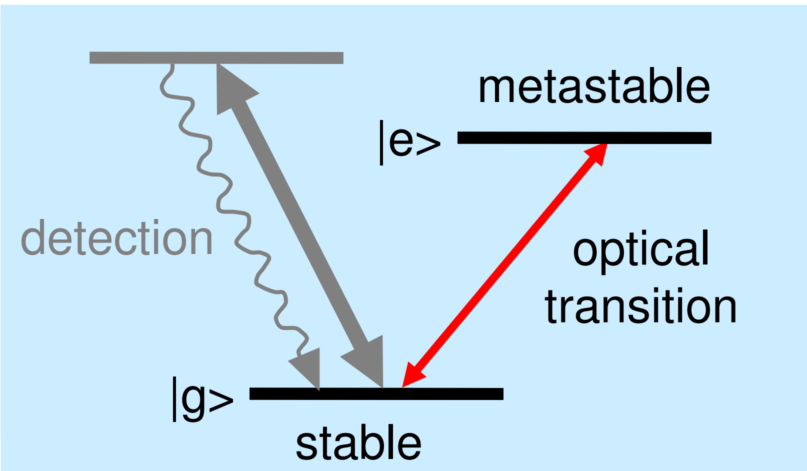


Qubit levels: Hyperfine levels of ground state $S_{1/2}$

Qubit transition: Raman transition between hyperfine levels

Disadvantage of “optical” quantum bits

Ions with optical transition to metastable level: $^{40}Ca^{+}$, $^{88}Sr^{+}$, $^{172}Yb^{+}$

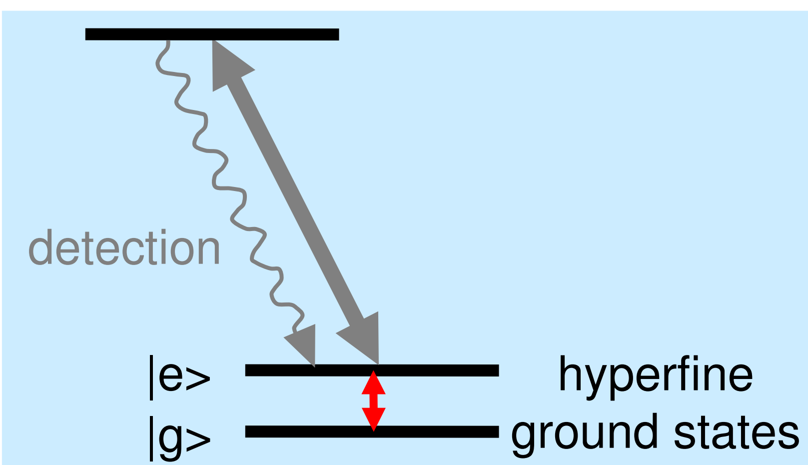


„optical qubit“

qubit manipulation requires ultrastable laser

Coherence time limited by laser linewidth

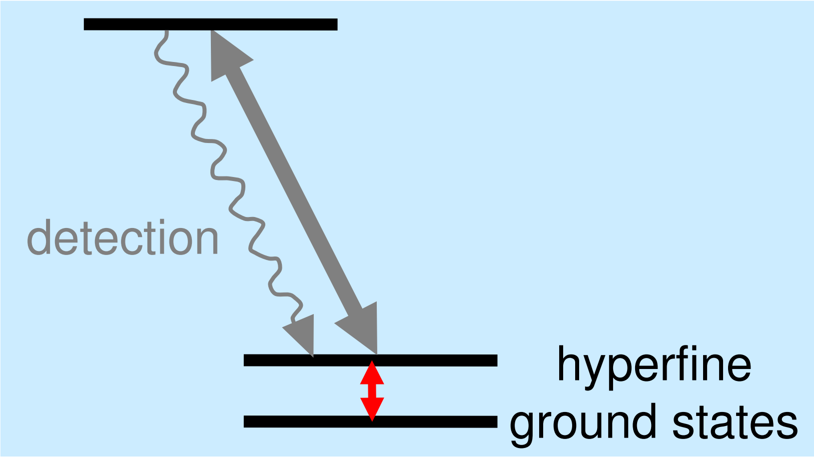
Ions with hyperfine structure: $^{9}Be^{+}$, $^{43}Ca^{+}$, $^{111}Cd^{+}$,…



„hyperfine qubit“

qubit manipulation with microwaves or lasers

Hyperfine qubit + microwaves



Single qubit manipulation with microwaves:

- well established technique (NMR),
- long coherence times

Problems:

- Addressing of single qubit in ion string difficult
- Coupling between internal and motional states:

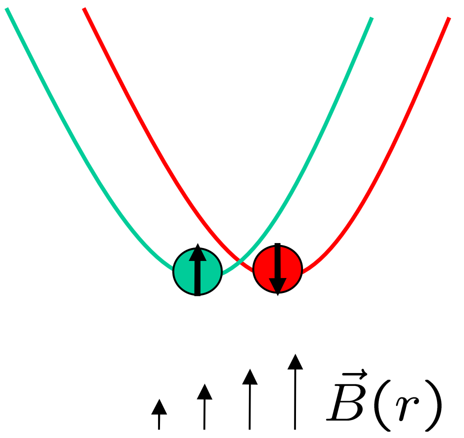
$$\eta \propto \frac{\text{size of ground state}}{\text{wavelength}} \ll 1 \qquad (\eta \propto 10^{-7})$$

Coupling constant neglegible !

Hyperfine qubit + microwaves + magn. field gradient ?

(F. Mintert, C. Wunderlich, PRL 87, 257904 (2001))

Apply magnetic field gradient to obtain state-dependent potential:



$$U_{field}(m_j, x) = m_j g_j \mu_B \frac{dB_z}{dx} x$$

$$\longrightarrow \qquad H = \hbar\nu(a^\dagger a + \tfrac{1}{2}) + \hbar\gamma\sigma_z(a + a^\dagger)$$

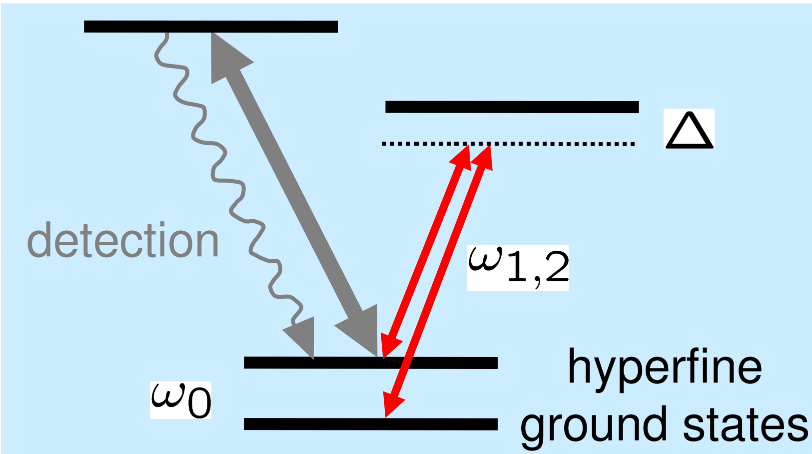
Coupling constant: $\gamma \propto \dfrac{\mu_B B' x_0}{\hbar}$ (x_0: size of ground state)

Example:

$$\left.\begin{aligned} \gamma &= (2\pi)10\ \mathrm{kHz} \\ x_0 &= 10\ \mathrm{nm} \end{aligned}\right\} \quad B' = 1\ \mathrm{T/cm}$$

requires strong field gradients

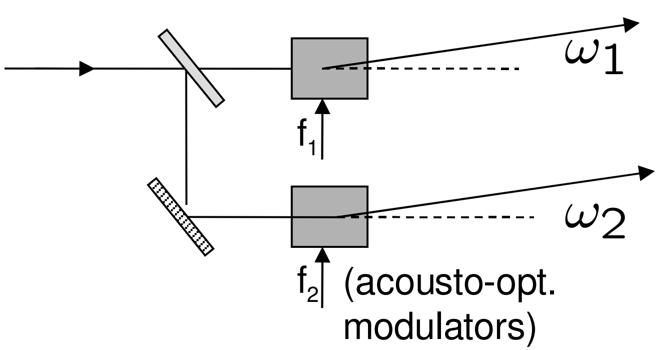
Hyperfine qubit + Raman transitions



Coupling between hyperfine states by Raman transition

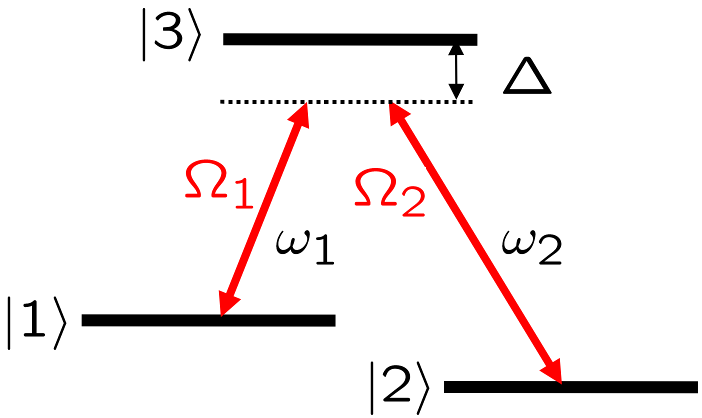
$$\omega_2 - \omega_1 = \omega_0 \pm n\nu, \ \ n = 0, 1$$

If both Raman beams are derived from the same laser, the laser line width Γ_L is not important as long as $\Gamma_L << \Delta$.



Raman beam generation

Raman transitions: Three-level system



Laser frequencies $\omega_{1,2}$

Rabi frequencies $\Omega_{1,2}$

Detuning from upper state $\Delta \gg \Omega_{1,2}$

$$H_I = \hbar\Delta|3\rangle\langle 3| + \frac{\hbar\Omega_1}{2}(|1\rangle\langle 3| + |3\rangle\langle 1|) + \frac{\hbar\Omega_2}{2}(|2\rangle\langle 3| + |3\rangle\langle 2|)$$

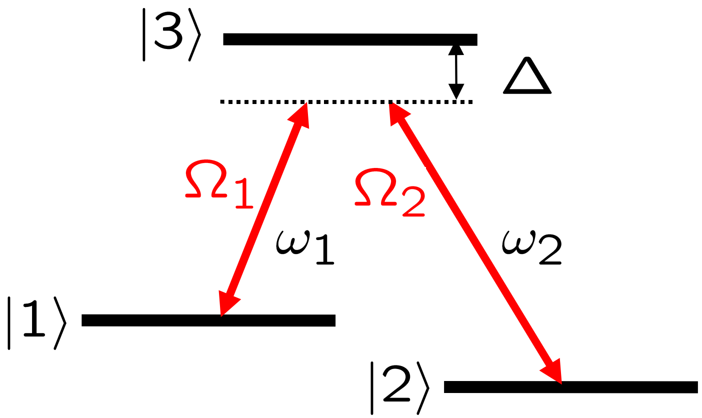
After adiabatic elimination of upper level:

Coupling between ground states

$$\Omega_{Raman} = \frac{\Omega_1\Omega_2}{2\Delta}$$

Rabi frequency of Raman process

Raman transitions: Further effects



Laser frequencies $\omega_{1,2}$

Rabi frequencies $\Omega_{1,2}$

Detuning from upper state $\Delta \gg \Omega_{1,2}$

2. The energies of states |1>,|2> are light-shifted because of the interaction with |3> by

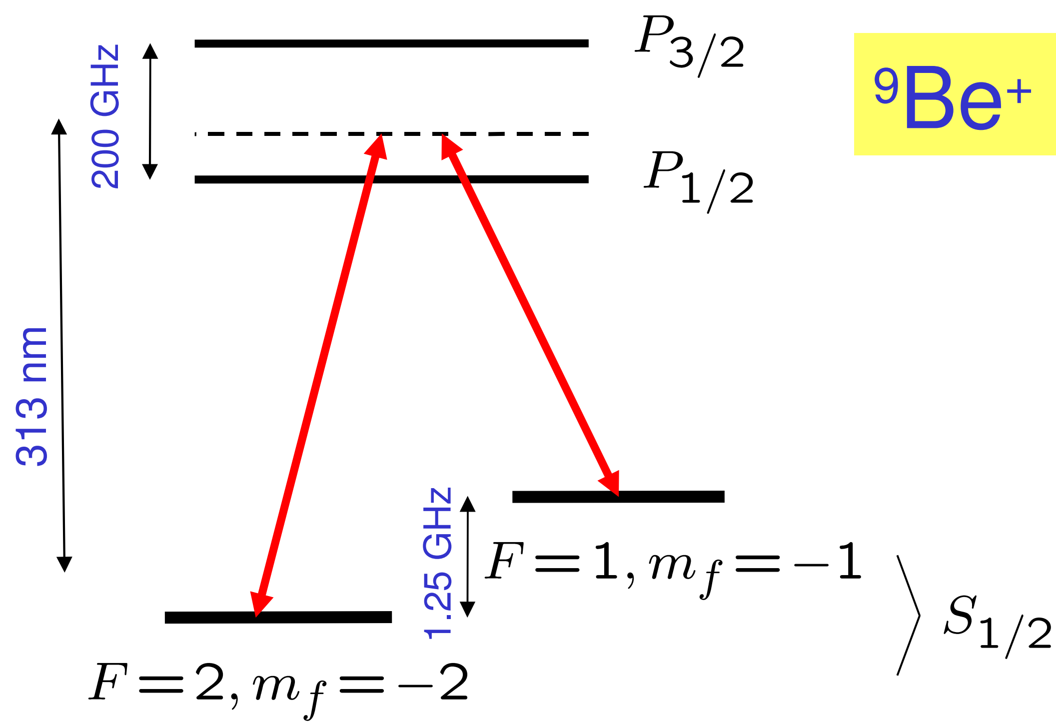
$$\delta_{1,3} = \frac{\Omega_{1,3}^2}{4\Delta}$$

In case of unequal light shifts, the transition frequency between |1> and |2> depends on the laser intensity. (laser intensity noise -> decoherence)

3. Population in state |3> : $\rho_{33} \approx \left(\frac{\Omega}{2\Delta}\right)^2 \qquad (\Omega_1 = \Omega_3)$

photon scattering rate $R = \Gamma \rho_{33}$ $\qquad \frac{\Omega_{Raman}}{R} = \frac{2\Delta}{\Gamma}$ $\qquad$ choose large detuning

Raman transitions in real ions



$^9Be^+$ (used in Wineland group, Boulder)

Nuclear spin $I = 3/2$

$\Gamma_{P_{1/2}} = 19.4$ MHz

Raman beams tuned to frequency within $P_{1/2}$ fine-structure

$\Delta \ll \Delta_{\text{fine structure}}$: Constructive interference between $P_{1/2}$ and $P_{3/2}$ virtual levels
strong Raman coupling

$\Delta \gg \Delta_{\text{fine structure}}$: Destructive interference between $P_{1/2}$ and $P_{3/2}$ virtual levels
weak Raman coupling, but longer hyperfine coherence times
(only Raman scattering destroys hyperfine coherence,
R. Ozeri et al, PRL **95**, 030403 (2005))

Raman transitions in real ions

$^{111}Cd^{+}$

Monroe group
Univ. Michigan

$I = 1/2 \longrightarrow$ simple level structure

$\Delta_{hfs} = 14.5$ GHz (Raman beam generation technically challenging)

P.Lee et al, Opt. Lett. **28**, 1582 (2003)

$^{43}Ca^{+}$

Steane group
Oxford

Innsbruck

$I = 7/2 \longrightarrow$ many levels

more convenient laser wave lengths

A. Steane, Appl. Phys. B 64, 623 (1997)

see also: D. Wineland et al. Phil. Trans. R. Soc. Lond. **A 361**, 1349 (2003)

Raman transitions: motional sidebands

$$H_{int} = \frac{\hbar\Omega}{2}\sigma_+\{\mathbb{1}+i\eta(e^{-i\nu t}a+e^{i\nu t}a^\dagger)\}e^{-i\delta t+i\phi} + h.c.$$

Same Hamiltonian as for two-level system, with

$\Omega = \frac{\Omega_1\Omega_2}{2\Delta}$ Raman Rabi frequency

$\delta = (\omega_2 - \omega_1) - \omega_{hfs}$ Raman detuning from hyperfine transition

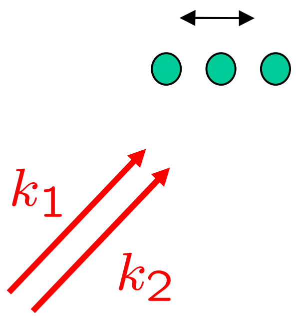
$\phi = \phi_2 - \phi_1$ difference of Raman laser phases

$\eta = (\vec{k_2} - \vec{k_1})\vec{n}_{mode}\, x_0$ effective Lamb Dicke parameter

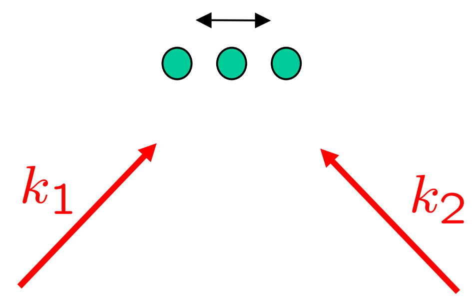
Raman transitions: motional sidebands

$\eta = (\vec{k_2} - \vec{k_1})\vec{n}_{mode}\, x_0$ effective Lamb Dicke parameter

copropagating Raman beams for excitation of carrier transitions (no sideband transitions)



counterpropagating Raman beams for efficient excitation of sideband transitions
or
beams from different directions



Decoherence issues

hyperfine (Raman) qubit	optical qubit
relative laser path length fluctuations	laser frequency noise
(stable setup, insensitive gate operations)	(ultrastable laser)
Raman photons scattered from virtual level	decay of metastable state
(choose ion with big fine structure)	(long-lived state)

magnetic field fluctuations

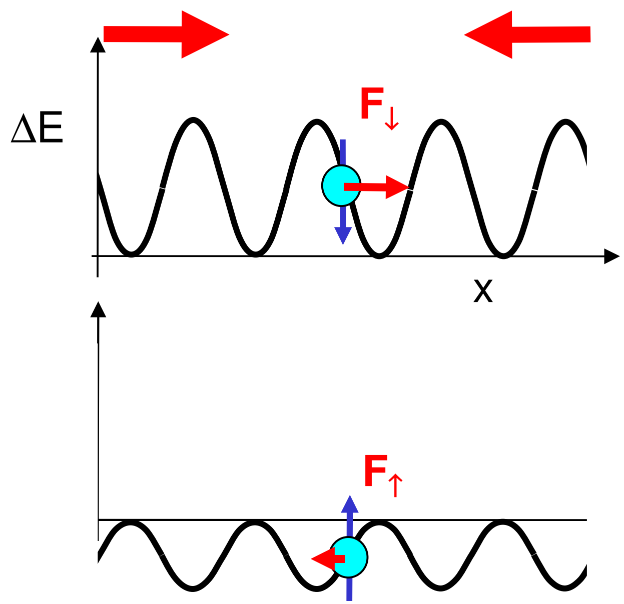
(screen magnetic fields, use field-independent transition)

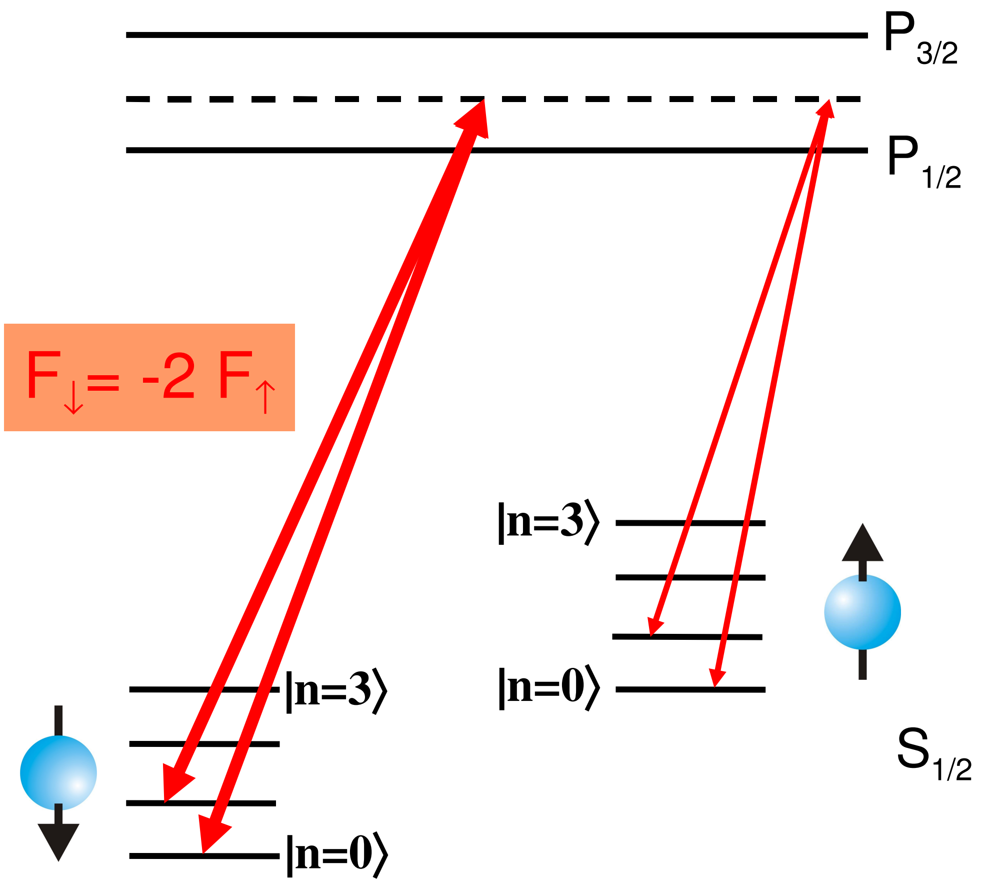
fluctuations of control parameters

Entangling interactions: controlled phase gate

Use Raman beams that couple the motional states (but not internal states)

Raman beams form (moving) standing wave: spatial light shifts

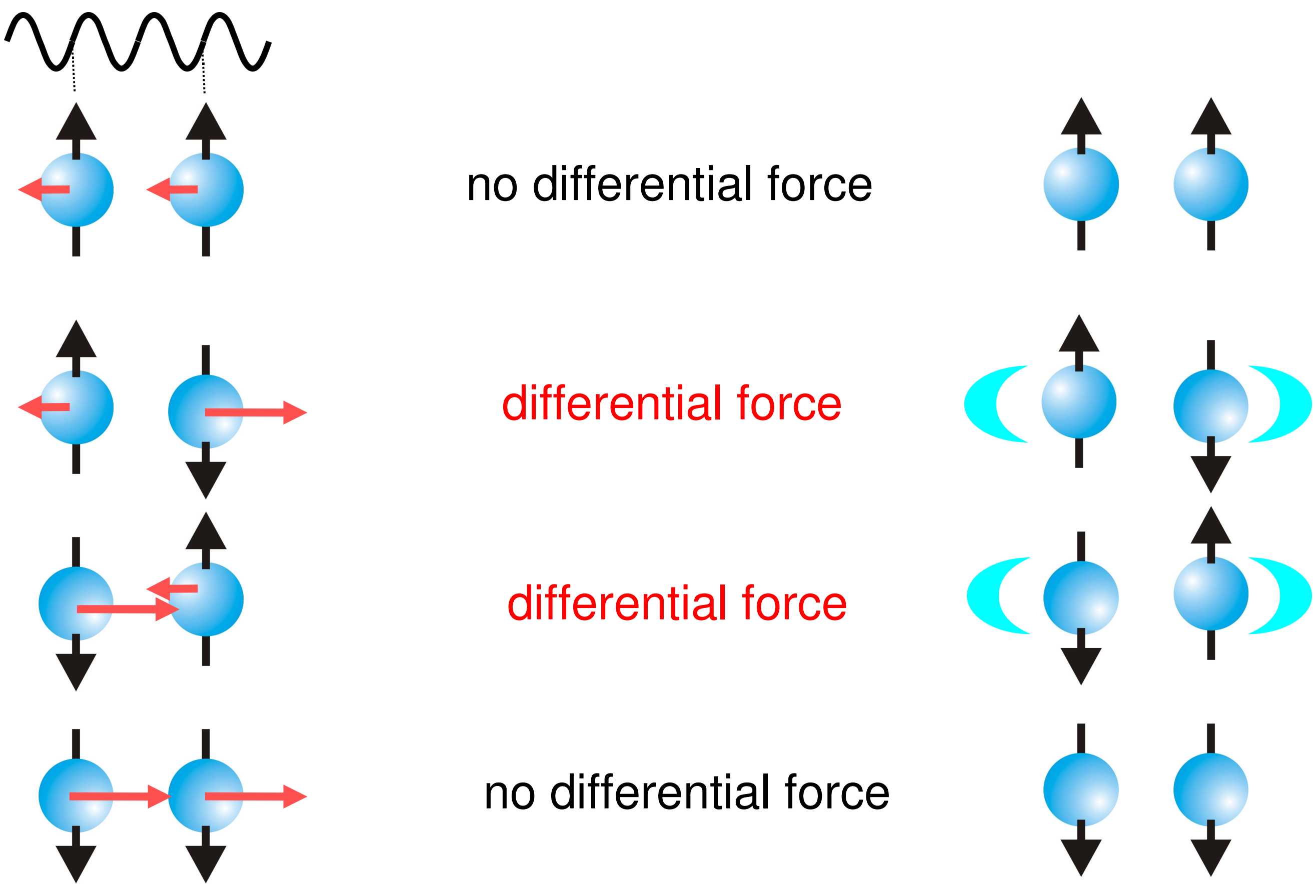




$F_\downarrow = -2\, F_\uparrow$

State dependent
optical dipole force

Stretch mode excitation



$$H(t) = (\alpha(t)a + \alpha^*(t)a^\dagger)(\sigma_z^{(1)} - \sigma_z^{(2)})$$

Controlled phase gate

$$H(t) = (\alpha(t)\hat{a} + \alpha^*(t)\hat{a}^\dagger)\,(\sigma_z^{(1)} - \sigma_z^{(2)})$$

$$= (\alpha_t\hat{a} + \alpha_t^*\hat{a}^\dagger)\,\mathcal{O} \qquad\qquad \mathcal{O} = \sigma_z^{(1)} - \sigma_z^{(2)}$$

$$[H(t_2), H(t_1)] = (\alpha_{t_2}\alpha_{t_1}^* - \alpha_{t_2}^*\alpha_{t_1})\mathcal{O}^2 \qquad\qquad \mathcal{O}^2 = 2(\mathbb{1} - \sigma_z^{(1)}\sigma_z^{(2)})$$

$$e^{-iH(t_2)\Delta t}e^{-iH(t_1)\Delta t} = e^{-iH_2\Delta t - iH_1\Delta t + \frac{1}{2}[H_2,H_1](\Delta t)^2}$$

$$e^{-iH(t_3)\Delta t}e^{-iH(t_2)\Delta t}e^{-iH(t_1)\Delta t} = e^{-i\sum_j H_j\Delta t + \frac{1}{2}\sum_{i<j}[H_j,H_i](\Delta t)^2}$$

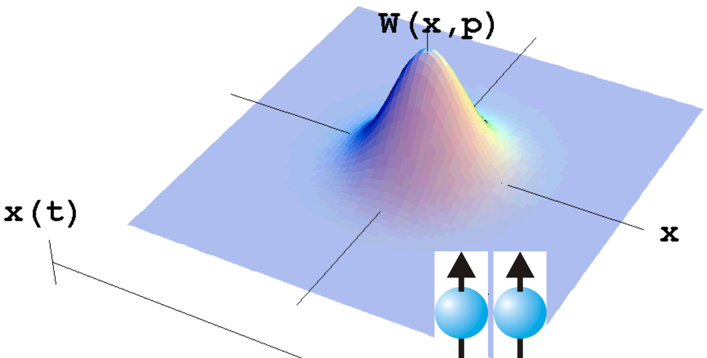
$$= e^{-i\sum_j H_j\Delta t + \frac{1}{2}\mathcal{O}^2\sum_{i<j}(\alpha_{t_j}\alpha_{t_i}^* - \alpha_{t_j}^*\alpha_{t_i})(\Delta t)^2}$$

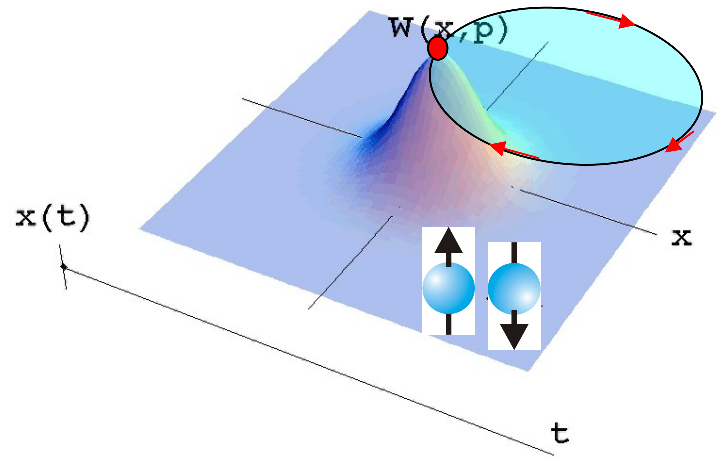
Time evolution operator:

$$U(t) = e^{-i\int_0^t H(t')dt' + \frac{1}{2}\mathcal{O}^2(\int_0^t dt'\alpha(t')\int_0^{t'} dt''\alpha^*(t'') - h.c.)}$$

Choose α(t‘) such that $\int_0^t \alpha(t')dt' = 0 \longrightarrow$ $\boxed{U = e^{-i\theta\sigma_z^{(1)}\sigma_z^{(2)}}}$

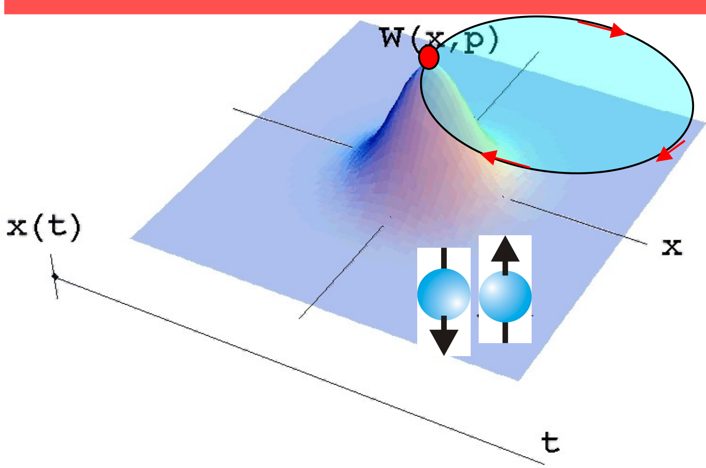
Phase space picture





Boulder group :
Gate fidelity: 97%
Gate time: 7 µs (ca. 25/ν_{COM})

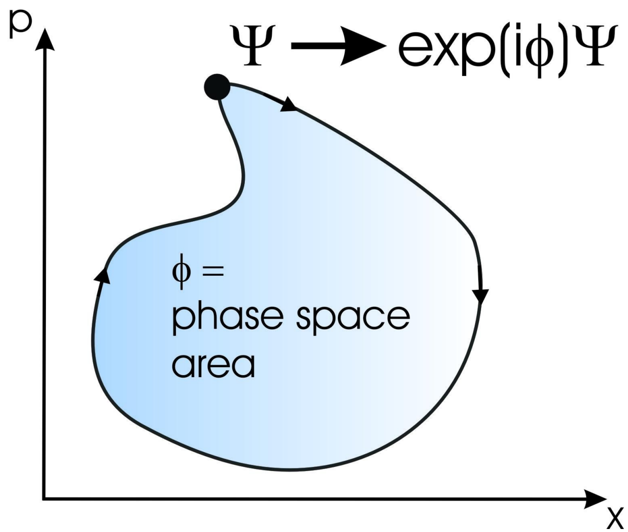
D. Leibfried *et al.*, Nature **422**, 414 (2003)
Theory: Milburn, Fortschr. Phys. **48**, 9(2000)
Sørensen&Mølmer



$$|\downarrow\downarrow\rangle \rightarrow |\downarrow\downarrow\rangle$$
$$|\downarrow\uparrow\rangle \rightarrow e^{i\phi}|\downarrow\uparrow\rangle$$
$$|\uparrow\downarrow\rangle \rightarrow e^{i\phi}|\uparrow\downarrow\rangle$$
$$|\uparrow\uparrow\rangle \rightarrow |\uparrow\uparrow\rangle$$

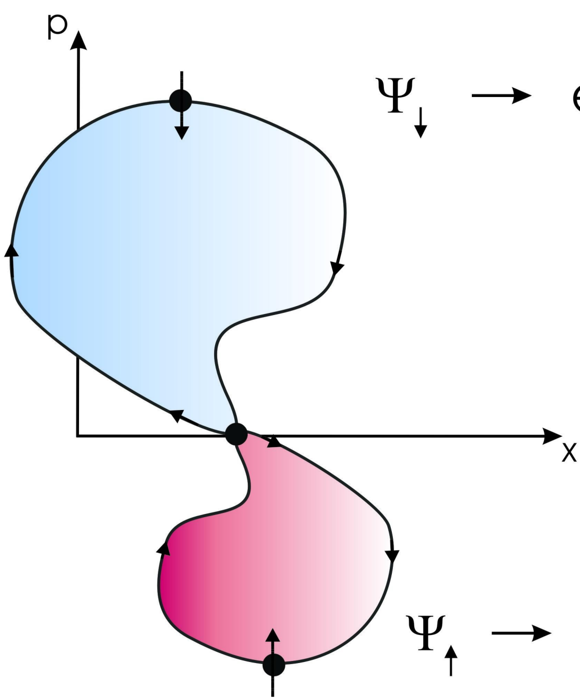
Geometric phase gate

$\Psi \longrightarrow \exp(i\phi)\Psi$



1) coherent displacement along closed path will shift phase of the quantum state, phase independent of details like speed of traversal, etc.

2) the sign and magnitude of coherent displacements can be made internal-state dependent (see e.g. Science **272**, 1131 (1996))



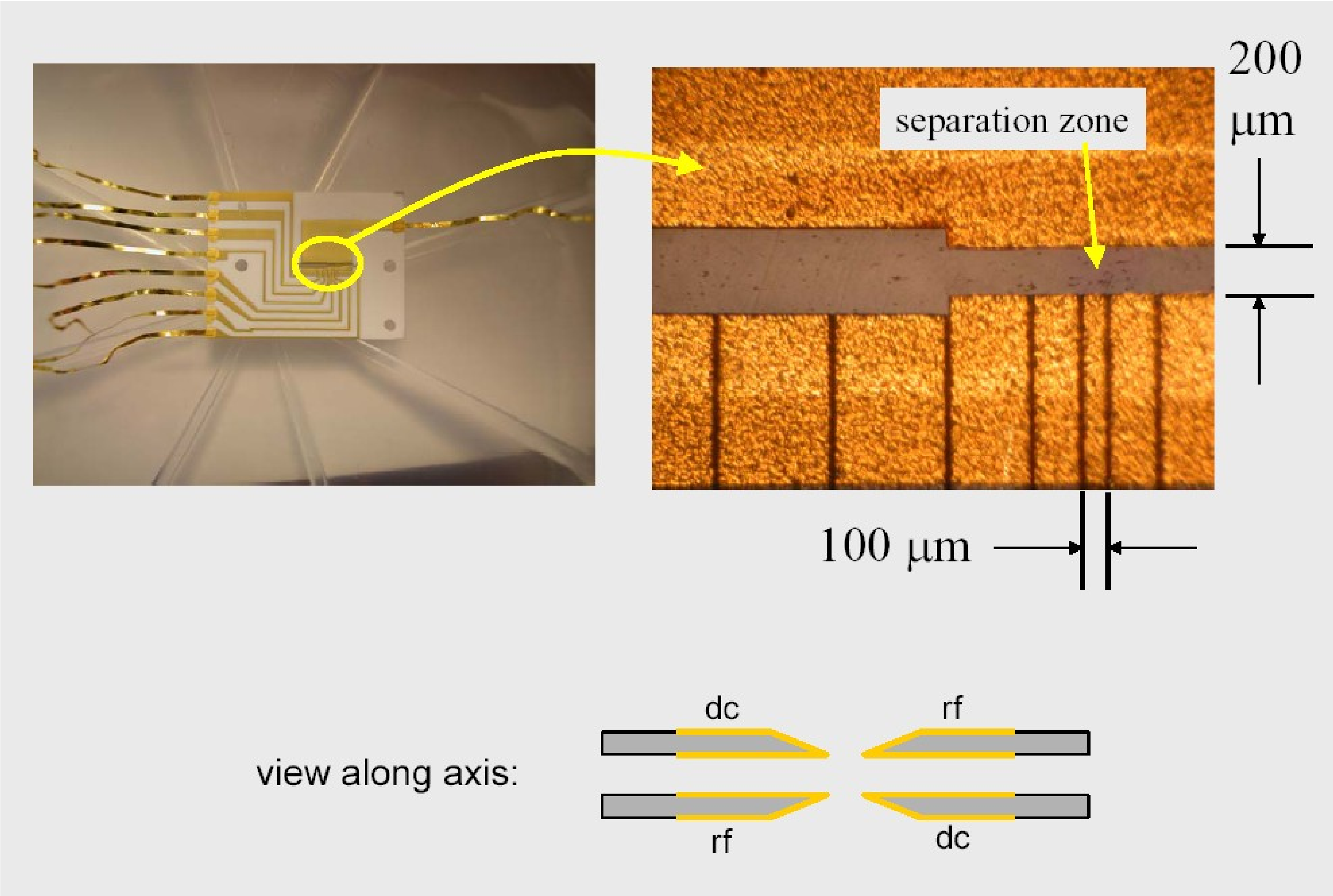
$\Psi_{\downarrow} \rightarrow \exp(i\phi)\, \Psi_{\downarrow}$

$\Psi_{\uparrow} \rightarrow \exp(i\phi/4)\, \Psi_{\uparrow}$

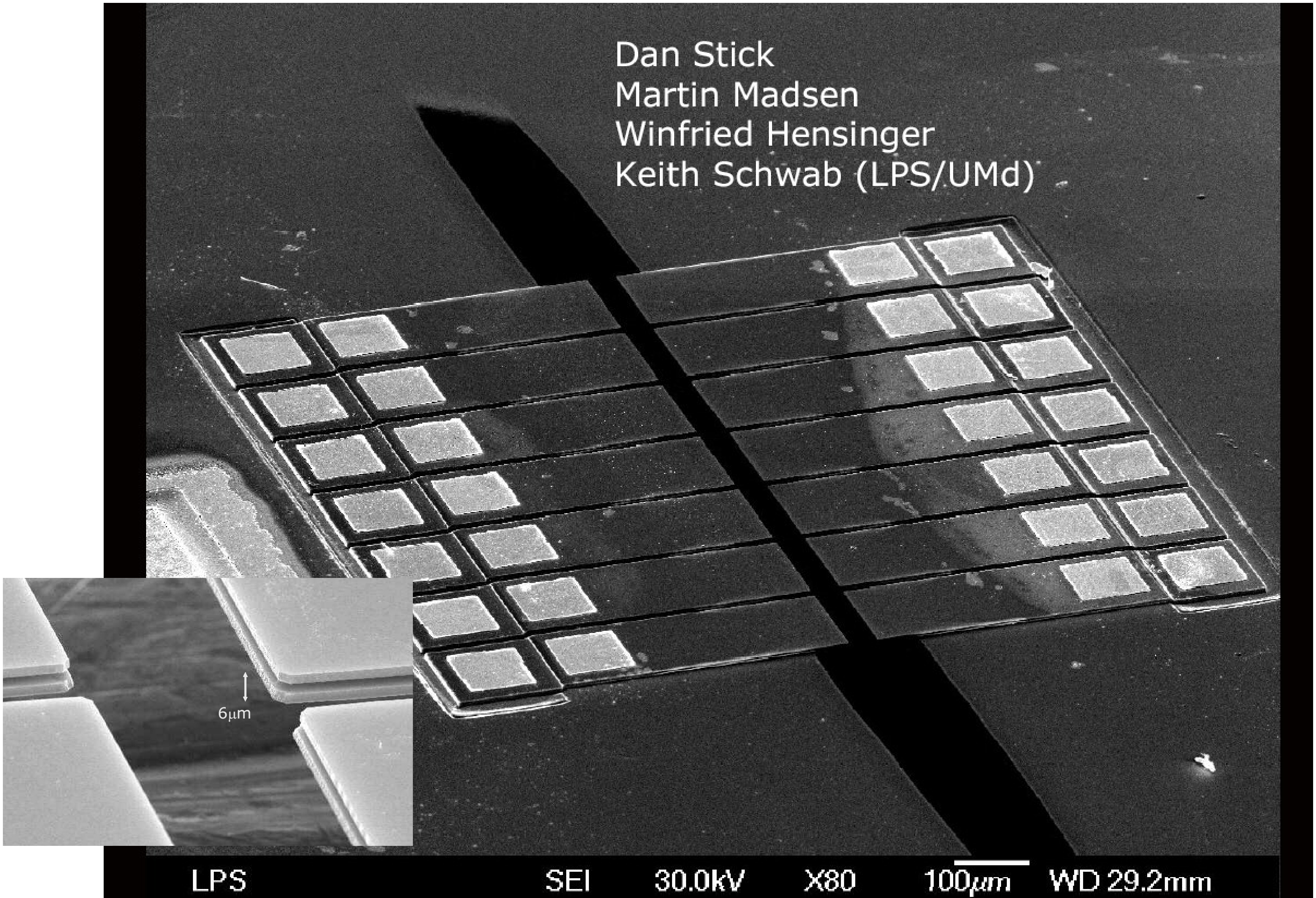
-no ground state cooling
-no individual addressing
-robust against "small" deformations of the path
-relative phase of successive displacements irrelevant

G. J. Milburn *et al.*, Fortschr. Physik 48, 801 (2000).
X. Wang *et al.*, Phys. Rev. Lett. 86, 3907 (2001).

D. Wineland (NIST) : Alumina / gold trap

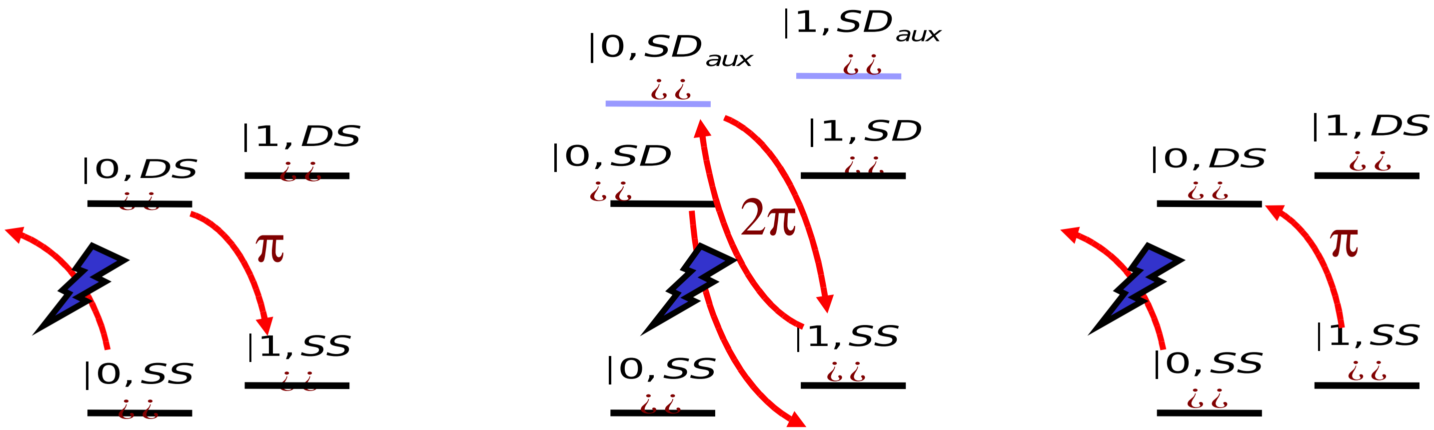


C. Monroe (Michigan) : GaAs-GaAlAs trap



Ideal Cirac-Zoller phase gate

(J. I. Cirac und P. Zoller, PRL **74** 4091 (1995))



qubits		red sideband π-pulse on ion #1		red sideband 2π-pulse on ion #2		red sideband π-pulse on ion #1
00	$\lvert 0,SS\rangle$	$\rightarrow$	$\lvert 0,SS\rangle$	$\rightarrow$	$\lvert 0,SS\rangle$	$\rightarrow$ $\lvert 0,SS\rangle$
01	$\lvert 0,SD\rangle$	$\rightarrow$	$\lvert 0,SD\rangle$	$\rightarrow$	$\lvert 0,SD\rangle$	$\rightarrow$ $\lvert 0,SD\rangle$
10	$\lvert 0,DS\rangle$	$\rightarrow$	$i\lvert 1,SS\rangle$	$\rightarrow$	$-i\lvert 1,SS\rangle$	$\rightarrow$ $\lvert 0,DS\rangle$
11	$\lvert 0,DD\rangle$	$\rightarrow$	$i\lvert 1,SD\rangle$	$\rightarrow$	$i\lvert 1,SD\rangle$	$\rightarrow$ $-\lvert 0,DD\rangle$